ISSN 2075-9827 **e-ISSN 2313-0210**
Carpathian Math. Publ. 2024, **16** (2), 407–413
doi:10.15330/cmp.16.2.407-413

https://journals.pnu.edu.ua/index.php/cmp
Карпатськi матем. публ. 2024, Т.16, №2, С.407–413

The Waring-Girard formulas for symmetric polynomials on spaces ℓ_p

Handera-Kalynovska O.V., Kravtsiv V.V.✉

Classical Waring-Girard formulas gives a representation of elementary and complete symmetric polynomials through the power symmetric polynomials. In this paper, we propose some analogs of the Waring-Girard formulas in the case of spaces ℓ_p, where $1 \leq p < \infty$, and show an application of obtained formulas in combinatorics.

Key words and phrases: symmetric polynomial on a Banach space, Waring-Girard formula, algebraic bases, combinatorial relation.

Vasyl Stefanyk Precarpathian National University, 57 Shevchenka str., 76018, Ivano-Frankivsk, Ukraine
✉ Corresponding author
E-mail: olha.handera@pnu.edu.ua (Handera-Kalynovska O.V.),
viktoriia.kravtsiv@pnu.edu.ua (Kravtsiv V.V.)

Introduction

Let X be a topological linear space and S be a (semi)group of continuous operators on X. A mapping f on X is called S-*symmetric* (or just *symmetric*) if

$$f\big(\sigma(x)\big) = f(x)$$

for every $x \in X$ and $\sigma \in S$.

Symmetric polynomials and analytic functions on finite- and infinite-dimensional linear spaces appear in combinatorics and classical invariant theory (see, e.g., [12, 16]), nonlinear functional analysis [1–4, 7, 11], in applications to quantum physics [5], to informatics [17], and cryptography [6]. For infinite-dimensional Banach spaces, investigations of symmetric polynomials started by A. Nemirovskii and S. Semenov in [14] and M. González, R. Gonzalo, J.A. Jaramillo in [10]. In particular, in [14] the authors constructed algebraic bases of algebras of symmetric real-valued polynomials on real Banach spaces ℓ_p and $L_p[0,1]$ for $1 \leq p < \infty$. In [10], these results were generalized to Banach spaces with symmetric bases and to separable rearrangement invariant Banach spaces. The cases of ℓ_∞ and L_∞ were considered in [8, 9, 15].

Let us recall that a Schauder basis (e_n) of a complex Banach space X is *symmetric* if for every permutation (one-to-one map) $\sigma \in S_{\mathbb{N}}$, the basis $\big(e_{\sigma(n)}\big)$ is equivalent to (e_n), where $S_{\mathbb{N}}$ is the semigroup of all permutations on the set of all natural numbers $\mathbb{N}$. A mapping F on X is said to be *symmetric* if it is $S_{\mathbb{N}}$-symmetric, that is,

$$F(x_1, x_2, \ldots) = F\big(x_{\sigma(1)}, x_{\sigma(2)}, \ldots\big)$$

УДК 517.98
2020 *Mathematics Subject Classification:* 46J15, 46E10, 46E50.

This paper was supported by the Ministry of Education of Ukraine in the framework of the research project "Study of algebras generated by symmetric polynomial and rational mappings in Banach spaces", 0123U101791.

for each $\sigma \in S_{\mathbb{N}}$. A function $P\colon X \to \mathbb{C}$ is a polynomial of degree m if the restriction of P to any finite-dimensional subspace of X is a polynomial of several variables of degree $\leq m$ and there is a finite-dimensional subspace V of X such that the restriction of P to V is a polynomial of degree m. We denote by $\mathcal{P}_s(X)$ the algebra of all continuous symmetric polynomials on X.

It is known that polynomials

$$F_k(x) = \sum_{n=1}^{\infty} x_n^k, \quad k \in \mathbb{N},$$

form an algebraic basis in the algebra $\mathcal{P}_s(\ell_1)$ (see [10]). That is, for any polynomial $P \in \mathcal{P}_s(\ell_1)$ there is a unique polynomial of several complex variables $Q(t_1, \ldots, t_m)$ such that $P(x) = Q(F_1(x), \ldots, F_m(x))$. Polynomials F_k are called *power* symmetric polynomials. The algebraic basis is not unique. We will use also the following bases in $\mathcal{P}_s(\ell_1)$:

$$G_n(x) = \sum_{i_1 < \cdots < i_n} x_{i_1} \cdots x_{i_n}, \tag{1}$$

which is called the *basis of elementary symmetric polynomials* and

$$H_n(x) = \sum_{i_1 \leq \cdots \leq i_n} x_{i_1} \cdots x_{i_n}, \tag{2}$$

which is called the *basis of complete symmetric polynomials*. These bases are connected by known Newton formulas :

$$nG_n = \sum_{k=1}^{n} (-1)^{k-1} G_{n-k} F_k, \quad n \in \mathbb{N}, \tag{3}$$

$$nH_n = \sum_{k=1}^{n} H_{n-k} F_k, \quad n \in \mathbb{N}. \tag{4}$$

According to [10], in the general case $1 \leq p < \infty$, polynomials F_n, $n \geq \lceil p \rceil$, form an algebraic basis in $H_{bs}(\ell_p)$, where $\lceil p \rceil$ is the ceil of p. On the other hand, we can not define neither elementary or complete symmetric polynomials for any n by formulas (1) and (2) if $p > 1$, because the series in (1) and (2) does not converge for any n. However, setting in the Newton formulas (3) and (4) $F_k = 0$ for $k < p$, we can define elementary and complete symmetric polynomials on ℓ_p by

$$nG_n^{(p)} = \sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} (-1)^{k-1} F_k G_{n-k}^{(p)} + (-1)^{n-1} F_n,$$

and

$$nH_n^{(p)} = \sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} F_k H_{n-k}^{(p)} + F_n.$$

Here we assume that if $n - \lceil p \rceil < \lceil p \rceil$, then

$$\sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} (-1)^{k-1} F_k G_{n-k}^{(p)} = 0 \quad \text{and} \quad \sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} F_k G_{n-k}^{(p)} = 0.$$

Since all polynomials F_n, $n \geq p$, form an algebraic basis in $\mathcal{P}_s(\ell_p)$, from formulas above it follows that both sequences $\left(G_n^{(p)}\right)_{n\geq\lceil p\rceil}$ and $\left(H_n^{(p)}\right)_{n\geq\lceil p\rceil}$ form algebraic bases in $\mathcal{P}_s(\ell_p)$.

We denote by $\mathbb{Z}_+$ the set of all nonnegative integers. It is well-known that the elementary and complete symmetric polynomials can be expressed in terms of the power symmetric polynomials (see, e.g., [13, p. 6–7]) by the Waring-Girard formulas:

$$G_n = \sum_{\lambda_1+2\lambda_2+\ldots+n\lambda_n=n} \frac{(-1)^{n+(\lambda_1+\lambda_2+\ldots+\lambda_n)}}{1^{\lambda_1}\cdot 2^{\lambda_2}\cdot\ldots\cdot n^{\lambda_n}\lambda_1!\cdot\lambda_2!\cdot\ldots\cdot\lambda_n!}(F_1)^{\lambda_1}(F_2)^{\lambda_2}\ldots(F_n)^{\lambda_n} \tag{5}$$

and

$$H_n = \sum_{\lambda_1+2\lambda_2+\ldots+n\lambda_n=n} \frac{1}{1^{\lambda_1}\cdot 2^{\lambda_2}\cdot\ldots\cdot n^{\lambda_n}\lambda_1!\cdot\lambda_2!\cdot\ldots\cdot\lambda_n!}(F_1)^{\lambda_1}(F_2)^{\lambda_2}\ldots(F_n)^{\lambda_n}, \tag{6}$$

where $\lambda_j \in \mathbb{Z}_+, j = 1,\ldots,n$.

In this paper, we find some analogs of Waring-Girard formulas for the case of space ℓ_p, where $p > 1$, and propose an application to combinatorics.

1 Main results

Without loss of generality we can consider the spaces ℓ_p for positive integer numbers p. If p is not integer, then we can take $\lceil p\rceil$ instead of p. If we put $F_1 = 0,\ldots,F_{p-1} = 0$ to the formulas (5) and (6), we obtain the following analogs of the Waring-Girard formulas in the case of ℓ_p for $p \geq 1$:

$$G_n^{(p)} = \sum_{p\lambda_p+\ldots+n\lambda_n=n} \frac{(-1)^{n+(\lambda_p+\ldots+\lambda_n)}}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}\lambda_p!\cdot\ldots\cdot\lambda_n!}(F_p)^{\lambda_p}\ldots(F_n)^{\lambda_n} \tag{7}$$

and

$$H_n^{(p)} = \sum_{p\lambda_p+\ldots+n\lambda_n=n} \frac{1}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}\lambda_p!\cdot\ldots\cdot\lambda_n!}(F_p)^{\lambda_p}\ldots(F_n)^{\lambda_n}. \tag{8}$$

Theorem 1. *We have the following presentation for polynomials $G_n^{(p)}$ and $H_n^{(p)}$:*

$$G_n^{(p)} = \begin{cases} (-1)^{n+1}\frac{1}{n}\sum\limits_{i=1}^{\infty} x_i^n, & \text{if } p \leq n < 2p, \\ \sum\limits_{p\lambda_p+\ldots+n\lambda_n=n} \frac{(-1)^{n+(\lambda_p+\ldots+\lambda_n)}}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}} \sum\limits_{\substack{|k_r|=\lambda_r,\\ i_r^1<\ldots<i_r^s,\\ p\leq r\leq n}} \frac{1}{k_p!\ldots k_n!}\prod\limits_{j=p}^{n}\left(x_{i_j^1}^{k_j^1}\ldots x_{i_j^s}^{k_j^s}\right)^j, & \text{if } n \geq 2p, \end{cases} \tag{9}$$

and

$$H_n^{(p)} = \begin{cases} \frac{1}{n}\sum\limits_{i=1}^{\infty} x_i^n, & \text{if } p \leq n < 2p, \\ \sum\limits_{p\lambda_p+\ldots+n\lambda_n=n} \frac{1}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}} \sum\limits_{\substack{|k_r|=\lambda_r,\\ i_r^1<\ldots<i_r^s,\\ p\leq r\leq n}} \frac{1}{k_p!\ldots k_n!}\prod\limits_{j=p}^{n}\left(x_{i_j^1}^{k_j^1}\ldots x_{i_j^s}^{k_j^s}\right)^j, & \text{if } n \geq 2p, \end{cases} \tag{10}$$

where $|k_r| = k_r^1+\ldots+k_r^s, k_r! = k_r^1!\ldots k_r^s!, 1 \leq p \leq r \leq n$.

Proof. To prove formula (9), we substitute

$$F_k(x) = \sum_{n=1}^{\infty} x_n^k, \quad k \geq p,$$

to formula (7). In the case $p \leq n < 2p$ in (7) we have that $p\lambda_p + \ldots + n\lambda_n = n$ is true only if justa a single $\lambda_j = 1$ and other $\lambda_i = 0, i \neq j, i, j \in \{p, p+1, \ldots, n\}$, where $\lambda_j \in \mathbb{Z}_+$. In this case, we obtain

$$G_n^{(p)} = (-1)^{n+1}\frac{1}{n}F_n(x) = (-1)^{n+1}\frac{1}{n}\sum_{i=1}^{\infty} x_i^n.$$

In the case $n \geq 2p$, we have

$$G_n^{(p)} = \sum_{p\lambda_p+\ldots+n\lambda_n=n} \frac{(-1)^{n+\left(\lambda_p+\ldots+\lambda_n\right)}}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}\lambda_p!\cdot\ldots\cdot\lambda_n!}\left(\sum_{i=1}^{\infty} x_i^p\right)^{\lambda_p}\ldots\left(\sum_{i=1}^{\infty} x_i^n\right)^{\lambda_n}$$

$$= \sum_{p\lambda_p+\ldots+n\lambda_n=n} \frac{(-1)^{n+\left(\lambda_p+\ldots+\lambda_n\right)}}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}\lambda_p!\cdot\ldots\cdot\lambda_n!}$$

$$\times\left(\sum_{\substack{|k_p|=\lambda_p,\\ i_p^1<\ldots<i_p^s}} \frac{\lambda_p!}{k_p!}\left(x_{i_p^1}^{k_p^1}\ldots x_{i_p^s}^{k_p^s}\right)^p\right)\times\ldots\times\left(\sum_{\substack{|k_n|=\lambda_n,\\ i_n^1<\ldots<i_n^s}} \frac{\lambda_n!}{k_n!}\left(x_{i_n^1}^{k_n^1}\ldots x_{i_n^s}^{k_n^s}\right)^n\right)$$

$$= \sum_{p\lambda_p+\ldots+n\lambda_n=n} \frac{(-1)^{n+\left(\lambda_p+\ldots+\lambda_n\right)}}{p^{\lambda_p}\cdot\ldots\cdot n^{\lambda_n}} \sum_{\substack{|k_r|=\lambda_r,\\ i_r^1<\ldots<i_r^s,\\ p\leq r\leq n}} \frac{1}{k_p!\ldots k_n!}\left(x_{i_p^1}^{k_p^1}\ldots x_{i_p^s}^{k_p^s}\right)^p\ldots\left(x_{i_n^1}^{k_n^1}\ldots x_{i_n^s}^{k_n^s}\right)^n.$$

Formula (10) can be proved by the same way. □

Formulas (5) and (6) are useful in combinatorics. For example, the well-known combinatorial identity

$$\sum_{\lambda_1+2\lambda_2+\ldots+n\lambda_n=n} \frac{(-1)^{n+(\lambda_1+\lambda_2+\ldots+\lambda_n)}}{1^{\lambda_1}\cdot 2^{\lambda_2}\cdot\ldots\cdot n^{\lambda_n}\lambda_1!\cdot\lambda_2!\cdot\ldots\cdot\lambda_n!} = 0, \quad n > 1,$$

can be obtained if we compute $G_n(e_1)$ using (5), where $e_1 = (1, 0, 0, \ldots)$. By the similar way, we can get some new relations using (9).

Proposition 1. *Let* $\lambda_j \in \mathbb{Z}_+, j = 2, \ldots, n, n \geq 2$. *The following combinatorial identity*

$$\sum_{2\lambda_2+\ldots+n\lambda_n=n} \frac{(-1)^{n+(\lambda_2+\ldots+\lambda_n)}}{2^{\lambda_2}\cdot\ldots\cdot\cdot n^{\lambda_n}\lambda_2!\cdot\ldots\cdot\lambda_n!} = \frac{(-1)^{n+1}(n-1)}{n!} \tag{11}$$

is true.

Proof. To prove the identity, we compute $G_n^{(2)}(e_1)$ with $e_1 = (1, 0, 0, \ldots) \in \ell_2$, using (7) and the Newton formula. Indeed, for $p = 2$ we have

$$\sum_{2\lambda_2+\ldots+n\lambda_n=n} \frac{(-1)^{n+(\lambda_2+\ldots+\lambda_n)}}{2^{\lambda_2}\cdot\ldots\cdot n^{\lambda_n}\lambda_2!\cdot\ldots\cdot\lambda_n!} = G_n^{(2)}(e_1). \tag{12}$$

On the other hand, we claim that

$$G_n^{(2)}(e_1) = \frac{(-1)^{n+1}(n-1)}{n!}.$$

Let us prove it applying the mathematical induction to the analog of the Newton formula for $G_n^{(2)}$:

$$2G_n^{(2)} = \sum_{k=2}^{n-2}(-1)^{k-1}F_kG_{n-k}^{(2)} + (-1)^{n-1}F_n. \tag{13}$$

If $n = 2$ we have $2G_2^{(2)} = -F_2$. Thus, $G_2^{(2)}(e_1) = -\frac{1}{2}$. Let us assume that the required identity is true for every $2 \leq m < n$. That is,

$$G_{m-1}^{(2)}(e_1) = \frac{(-1)^m(m-2)}{(m-1)!}.$$

From formula (13) and the induction assumption we obtain

$$nG_n^{(2)}(e_1) = -\frac{(-1)^{n-1}(n-3)}{(n-2)!} + \frac{(-1)^{n-2}(n-4)}{(n-3)!} - \ldots + (-1)^{n-1}\left(-\frac{1}{2}\right) + (-1)^{n+1}$$

and so

$$(n-1)G_{n-1}^{(2)}(e_1) = -\frac{(-1)^{n-2}(n-4)}{(n-3)!} + \frac{(-1)^{n-3}(n-5)}{(n-4)!} - \ldots + (-1)^{n-2}\left(-\frac{1}{2}\right) + (-1)^n.$$

By adding the last formulas, we get

$$nG_n^{(2)}(e_1) + (n-1)G_{n-1}^{(2)}(e_1) = \frac{(-1)^n(n-3)}{(n-2)!}.$$

Hence,

$$nG_n^{(2)}(e_1) + (n-1)\frac{(-1)^n(n-2)}{(n-1)!} = \frac{(-1)^n(n-3)}{(n-2)!}.$$

From here we can get

$$nG_n^{(2)}(e_1) = -(n-1)\frac{(-1)^n(n-2)}{(n-1)!} + \frac{(-1)^n(n-3)}{(n-2)!} = \frac{(-1)^{(n+1)}}{(n-2)!}.$$

So

$$G_n^{(2)}(e_1) = \frac{(-1)^{n+1}(n-1)}{n!}.$$

Substituting $G_n^{(2)}(e_1)$ to (12), we obtain the required equality (11). □

Clearly, the correspondence $G_n \rightsquigarrow G_n^{(p)}$ and $H_n \rightsquigarrow H_n^{(p)}$ is not a unique way to extend G_n and H_n to ℓ_p for $n \geq p$. In the general case, we can substitute $F_1 \equiv a_1, F_2 \equiv a_2, \ldots, F_{p-1} \equiv a_{p-1}$ in formulas (5) and (6) for a given $a = (a_1, \ldots, a_{p-1}) \in \mathbb{C}^{p-1}$. Let $z = \big(z_1, \ldots, z_{p-1}\big)$ be a vector in $\mathbb{C}^{p-1}$ such that $F_1(z) = a_1, \ldots, F_{p-1}(z) = a_{p-1}$. Such a vector exists and unique for a given

a up to permutations of coordinates. Then we can define new symmetric polynomials $G_n^{(p,a)}$ and $H_n^{(p,a)}$, $n \geq \lceil p \rceil$, on ℓ_p by

$$nG_n^{(p,a)}(x) = \sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} (-1)^{k-1} F_k(x) G_{n-k}^{(p,a)}(x) + (-1)^{n-1} F_n(x) + \sum_{k<\lceil p \rceil} (-1)^{k-1} F_k(z) G_{n-k}^{(p,a)}(x) + \sum_{k=n-\lceil p \rceil+1}^{n} (-1)^{k-1} F_k(x) G_{n-k}(z),$$

and

$$nH_n^{(p,a)}(x) = \sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} F_k(x) H_{n-k}^{(p,a)}(x) + F_n(x) + \sum_{k<\lceil p \rceil} F_k(z) H_{n-k}^{(p,a)}(x) + \sum_{k=n-\lceil p \rceil+1}^{n} F_k(x) H_{n-k}(z).$$

Note, that polynomials $G_n^{(p,a)}$ and $H_n^{(p,a)}$ are not homogeneous if $a \neq 0$ but still form algebraic bases. It is easy to check that

$$G_n^{(p,a)} = \sum_{1\lambda_1+\ldots+n\lambda_n=n} \frac{(-1)^{n+\left(\lambda_p+\ldots+\lambda_n\right)} a_1^{\lambda_1} a_2^{\lambda_2} \ldots a_{p-1}^{\lambda_{p-1}}}{1^{\lambda_1} \cdot \ldots \cdot n^{\lambda_n} \lambda_1! \cdot \ldots \cdot \lambda_n!} \left(F_p\right)^{\lambda_p} \ldots \left(F_n\right)^{\lambda_n}$$

and

$$H_n^{(p,a)} = \sum_{1\lambda_1+\ldots+n\lambda_n=n} \frac{a_1^{\lambda_1} a_2^{\lambda_2} \ldots a_{p-1}^{\lambda_{p-1}}}{1^{\lambda_1} \cdot \ldots \cdot n^{\lambda_n} \lambda_1! \cdot \ldots \cdot \lambda_n!} \left(F_p\right)^{\lambda_p} \ldots \left(F_n\right)^{\lambda_n}.$$

References

[1] Alencar R., Aron R., Galindo P., Zagorodnyuk A. *Algebra of symmetric holomorphic functions on ℓ_p*. Bull. Lond. Math. Soc. 2003, **35** (1), 55–64. doi:10.1112/S0024609302001431

[2] Aron R.M., Falcó J., Maestre M. *Separation theorems for group invariant polynomials*. J. Geom. Anal. 2018, **28**, 393–404. doi:10.1007/s12220-017-9825-0

[3] Aron R., Galindo P., Pinasco D., Zalduendo I. *Group-symmetric holomorphic functions on a Banach space*. Bull. Lond. Math. Soc. 2016, **48** (5), 779–796. doi.org:10.1112/blms/bdw043

[4] Chernega I., Galindo P., Zagorodnyuk A. *Some algebras of symmetric analytic functions and their spectra*. Proc. Edinb. Math. Soc.(2) 2012, **55** (1), 125–142. doi:10.1017/S0013091509001655

[5] Chernega I., Martsinkiv M., Vasylyshyn T., Zagorodnyuk A. *Applications of Supersymmetric Polynomials in Statistical Quantum Physics*. Quantum Rep. 2023, **5** (4), 683–697. doi:10.3390/quantum5040043

[6] Chopyuk Y., Vasylyshyn T., Zagorodnyuk A. *Rings of Multisets and Integer Multinumbers*. Mathematics 2022, **10** (5), 778. doi:10.3390/math10050778

[7] Falcó J., García D., Jung M., Maestre M. *Group-invariant separating polynomials on a Banach space*. Publ. Mat. 2022, **66** (1), 207–233. doi:10.5565/PUBLMAT6612209

[8] Galindo P., Vasylyshyn T., Zagorodnyuk A. *The algebra of symmetric analytic functions on L_∞*. Proc. Roy. Soc. Edinburgh Sect. A 2017, **147** (4), 743–761. doi:10.1017/S0308210516000287

[9] Galindo P., Vasylyshyn T., Zagorodnyuk A. *Analytic structure on the spectrum of the algebra of symmetric analytic functions on L_∞*. RACSAM 2020, **114**, article number 56. doi:10.1007/s13398-020-00791-w

[10] González M., Gonzalo R., Jaramillo J.A. *Symmetric polynomials on rearrangement-invariant function spaces*. J. Lond. Math. Soc. (2) 1999, **59** (2), 681–697. doi:10.1112/S0024610799007164

[11] Kravtsiv V. *Block-Supersymmetric Polynomials on Spaces of Absolutely Convergent Series*. Symmetry 2024, **16** (2), 179. doi:10.3390/sym16020179

[12] Macdonald I.G. Symmetric Functions and Orthogonal Polynomials. Univ. Lecture Ser., **12**, AMS, Providence, RI, 1997.

[13] Macmahon Percy A. Combynatory analysis, Volume I. Cambridge University Press., 1915.

[14] Nemirovskii A., Semenov S. *On polynomial approximation of functions on Hilbert space*. Mat. USSR-Sb 1973, **21**, 255–277. doi:10.1070/SM1973v021n02ABEH002016

[15] Vasylyshyn T. *Algebras of Symmetric and Block-Symmetric Functions on Spaces of Lebesgue Measurable Functions*. Carpathian Math. Publ. 2024, **16**, 174–189. doi:10.15330/cmp.16.1.174-189

[16] Weyl H. The classical group: their invariants and representations. In: Princeton Landmarks in Mathematics and Physics. Princeton University Press: Princeton, New Jersey, 1973.

[17] Zagorodnyuk A., Baziv N., Chopyuk Y., Vasylyshyn T., Burtnyak I., Kravtsiv V. *Symmetric and Supersymmetric Polynomials and Their Applications in the Blockchain Technology and Neural Networks*. In Proceedings of 2023 IEEE World Conference on Applied Intelligence and Computing (AIC), Sonbhadra, India, 2023, 508–513, doi:10.1109/AIC57670.2023.10263936

Received 30.05.2024

Гандера-Калиновська О.В., Кравців В.В. *Формули Варінга-Гірарда для симетричних поліномів на просторах* ℓ_p // Карпатські матем. публ. — 2024. — Т.16, №2. — C. 407–413.

Класичні формули Варінга-Гірарда дають зображення елементарних та повних симетричних поліномів через степеневі симетричні поліноми. У цій статті отримано аналоги формул Варінга-Гірарда для випадку просторів ℓ_p, де $1 \leq p < \infty$, та запропоновано застосування отриманих формул до комбінаторики.

Ключові слова і фрази: симетричний поліном на банаховому просторі, формула Варінга-Гірарда, алгебраїчний базис, комбінаторна тотожність.