



Certain solitons on α -cosymplectic manifolds

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In the present paper, we characterize some solitons such as η -Einstein soliton, η -Yamabe soliton and Ricci-Yamabe soliton on α -cosymplectic manifolds. Furthermore, we characterize 3-dimensional α -cosymplectic manifolds with gradient Ricci-Yamabe soliton. Finally, we construct an example.

Key words and phrases: Einstein soliton, η -Einstein soliton, η -Ricci soliton, gradient η -Ricci soliton, η -Yamabe soliton, α -cosymplectic manifold.

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Introduction

A Ricci soliton in a Riemannian manifold (M, g) is given by

$$\mathcal{L}_X g + 2S + 2\lambda g = 0,$$

where \mathcal{L} is the Lie-derivative, λ is a constant, S is the Ricci tensor and the vector field X is called the potential vector field. Ricci solitons are the self similar solutions of the Ricci flow equation

$$\frac{\partial}{\partial t} g_{ij} = -2R_{ij},$$

which was introduced by R.S. Hamilton [15]. Ricci solitons have also been studied by various authors such as [25–27] and many others.

As a generalization of Ricci solitons, the notion of η -Ricci solitons was introduced in the paper [11]. An η -Ricci soliton is given by

$$\mathcal{L}_X g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0,$$

where μ is a constant. If $\mu = 0$, then η -Ricci soliton reduces to Ricci soliton and if $\mu \neq 0$, then the η -Ricci soliton is called proper. η -Ricci solitons have been studied by various authors such as [3–6, 12, 17, 19, 22, 24] and many others. Like a Ricci soliton, an η -Ricci soliton is also called shrinking, steady or expanding according as $\lambda < 0$, $\lambda = 0$ or $\lambda > 0$, respectively. Recently, the second named author studied Ricci solitons on generalized Sasakian space forms [23].

Let (M, g) be a Riemannian manifold of dimension $2n + 1$. Then M is called an Einstein soliton [9] if there is a vector field X such that

$$\mathcal{L}_X g + 2S + (2\lambda - r)g = 0,$$

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where r is the scalar curvature of the Riemannian metric g . If the scalar curvature r is constant, then the Einstein soliton becomes a Ricci soliton.

As a generalization of Einstein soliton, the η -Einstein soliton is given by

$$\mathcal{L}_X g + 2S + (2\lambda - r)g + 2\mu\eta \otimes \eta = 0. \quad (1)$$

If the scalar curvature r is constant, then the η -Einstein soliton reduces to an η -Ricci soliton.

A Yamabe soliton [2] on a Riemannian or pseudo-Riemannian manifold (M, g) is defined by

$$\frac{1}{2}\mathcal{L}_X g = (r - \lambda)g.$$

A Yamabe soliton is said to be expanding, steady or shrinking if $\lambda < 0$, $\lambda = 0$ or $\lambda > 0$, respectively.

We define the notion of η -Yamabe soliton as

$$\frac{1}{2}\mathcal{L}_X g = (r - \lambda)g - \mu\eta \otimes \eta, \quad (2)$$

where λ and μ are constants. If $\mu = 0$, then the η -Yamabe soliton becomes a Yamabe soliton.

A Ricci-Yamabe soliton on Riemannian manifold (M, g) is the structure (g, X, λ, a, b) , defined [14] by

$$\mathcal{L}_V g + 2aS + (2\lambda - br)g = 0, \quad (3)$$

where \mathcal{L} is the Lie-derivative, S is the Ricci tensor, r is the scalar curvature and $\lambda, a, b \in \mathbb{R}$. If X is gradient of a smooth function f on M , then above notion is called gradient Ricci-Yamabe soliton and equation (3) reduces to

$$\nabla^2 f + aS = \left(\lambda - \frac{1}{2}br \right) g, \quad (4)$$

where $\nabla^2 f$ is the Hessian of f .

The Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) is said to be expanding, steady or shrinking according as λ is negative, zero or positive, respectively. A Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) is called an almost Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) if a, b and λ are smooth functions on M . A Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) is said to be a

- Ricci soliton (or gradient Ricci soliton) if $a = 1, b = 0$ (see [15]);
- Yamabe soliton (or gradient Yamabe soliton) if $a = 0, b = 1$ (see [16]);
- Einstein soliton (or gradient Einstein soliton) if $a = 1, b = -1$ (see [9]);
- ρ -Einstein soliton (or gradient ρ -Einstein soliton) if $a = 1, b = -2\rho$ (see [10]).

When $a \neq 0, 1$, Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) is proper.

We organize the paper as follows. In Section 2, we consider some solitons on α -cosymplectic manifolds. Next, in Section 3 we study gradient Ricci-Yamabe solitons on α -cosymplectic manifolds. Finally, in Section 4 we construct an example of a 5-dimensional α -cosymplectic manifold to verify our results.

1 Preliminaries

A $(2n + 1)$ -dimensional Riemannian manifold (M, g) is called an almost contact metric manifold if it admits a $(1, 1)$ -type tensor field ϕ , a vector field ξ , a 1-form η and a Riemannian metric g such that [8]

$$\begin{aligned} \phi^2 U &= -U + \eta(U)\xi, & \eta(\xi) &= 1, \\ g(\phi U, \phi V) &= g(U, V) - \eta(U)\eta(V), & g(\phi U, V) &= -g(U, \phi V) \end{aligned}$$

for any vector fields U, V of the manifold. The vector field ξ is called the Reeb or characteristic vector field.

An almost contact metric manifold is said to be normal if the Nijenhuis tensor of ϕ [8] vanishes. An almost Kenmotsu manifold is an almost contact metric manifold if $d\eta = 0$ and $d\Phi = 2\eta \wedge \Phi, \Phi(U, V) = g(U, \phi V)$.

A normal contact metric manifold (M, g) is said to be cosymplectic if the following relations

$$d\eta = 0, \quad d\Phi = 0$$

hold. Similarly,

$$(\nabla_U \phi)V = 0, \quad \nabla_U \xi = 0$$

for any U, V and ∇ is the Levi-Civita connection on M . Also, if

$$d\eta = 0, \quad d\Phi = 2\alpha\eta \wedge \Phi$$

are satisfied, then M is called an α -cosymplectic manifold [1, 20], α is a real number. Equivalently,

$$\begin{aligned} (\nabla_U \phi)V &= \alpha[g(\phi U, V)\xi - \eta(V)\phi U], \\ \nabla_U \xi &= \alpha[U - \eta(U)\xi]. \end{aligned} \tag{5}$$

In α -cosymplectic manifolds, it is well known [13, 21] that

$$\begin{aligned} R(U, V)\xi &= \alpha^2[\eta(U)V - \eta(V)U], & R(U, \xi)V &= \alpha^2[g(U, V)\xi - \eta(V)U], \\ R(U, \xi)\xi &= \alpha^2[\eta(U)\xi - U], & S(U, \xi) &= -2n\alpha^2\eta(U), \end{aligned}$$

where S is the Ricci tensor.

Proposition 1. *In a 3-dimensional α -cosymplectic manifold the Ricci tensor is defined [7] by*

$$S(U, V) = \left(\alpha^2 + \frac{r}{2}\right) g(U, V) - \left(3\alpha^2 + \frac{r}{2}\right) \eta(U)\eta(V). \tag{6}$$

Proposition 2. *In a 3-dimensional α -cosymplectic manifold the following relation*

$$\xi r = -2\alpha(6\alpha^2 + r) \tag{7}$$

holds.

Proof. Equation (6) implies

$$QU = \left(\alpha^2 + \frac{r}{2}\right) U - \left(3\alpha^2 + \frac{r}{2}\right) \eta(U)\xi.$$

Differentiating (7), after some calculations we obtain

$$(\nabla_V Q)U = \frac{1}{2}[(Vr)U - (Vr)\eta(U)\xi] - \alpha\left(3\alpha^2 + \frac{r}{2}\right)[g(U, V)\xi + \eta(U)V - 2\eta(U)\eta(V)\xi]. \tag{8}$$

Contracting V in the foregoing equation we have $\xi r = -2\alpha(6\alpha^2 + r)$. This completes the proof. □

2 Solitons on α -cosymplectic manifolds

Let us suppose that the α -cosymplectic manifold admits an η -Einstein soliton (g, ξ) . Then (1) implies

$$(\mathcal{L}_\xi g)(U, V) + 2S(U, V) + (2\lambda - r)g(U, V) + \mu\eta(U)\eta(V) = 0. \quad (9)$$

Now,

$$(\mathcal{L}_\xi g)(U, V) = g(\nabla_U \xi, V) + g(U, \nabla_V \xi).$$

Using (5) in the above equation, we get

$$(\mathcal{L}_\xi g)(U, V) = 2\alpha[g(U, V) - \eta(U)\eta(V)]. \quad (10)$$

Using (10) in (9), we obtain

$$S(U, V) = \left(\frac{r}{2} - \alpha - \lambda\right)g(U, V) + (\alpha - \mu)\eta(U)\eta(V). \quad (11)$$

Contracting the above equation, we infer

$$r = \frac{2[2n\alpha + (2n + 1)\lambda + \mu]}{2n - 1}, \quad (12)$$

which implies r is a constant. Then from (9), it reduces to an η -Ricci soliton. Hence we have the following result.

Theorem 1. *If an α -cosymplectic manifold admits an η -Einstein soliton (g, ξ) , then it is an η -Einstein manifold and hence it reduces to an η -Ricci soliton.*

If we take $\alpha = 0$ or 1 , then from (12) we obtain r is a constant. Thus we get the next assertion.

Corollary 1. *If a cosymplectic or Kenmotsu manifold admits an η -Einstein soliton (g, ξ) , then it is an η -Einstein manifold and hence it reduces to an η -Ricci soliton.*

If an α -cosymplectic manifold admits an η -Yamabe soliton (g, ξ) , then (2) implies

$$\frac{1}{2}(\mathcal{L}_\xi g)(U, V) = (r - \lambda)g(U, V) - \mu\eta(U)\eta(V). \quad (13)$$

Using (10) in (13) we infer

$$\alpha[g(U, V) - \eta(U)\eta(V)] = (r - \lambda)g(U, V) - \mu\eta(U)\eta(V).$$

Contracting the above equation, we get

$$r = \lambda + \frac{2n\alpha + \mu}{2n + 1},$$

which is a constant. Hence we conclude the following result.

Theorem 2. *If an α -cosymplectic manifold admits an η -Yamabe soliton, then the scalar curvature is constant.*

If we take $\alpha = 0$ or 1 , then results are similar to the above theorem. Hence we have the next assertion.

Corollary 2. *If a cosymplectic or Kenmotsu manifold admits an η -Yamabe soliton, then the scalar curvature is constant.*

If an α -cosymplectic manifold admits a proper Ricci-Yamabe soliton (g, ξ, λ, a, b) , then equation (3) implies

$$(\mathcal{L}_\xi g)(U, V) + 2aS(U, V) + (2\lambda - br)g(U, V) = 0.$$

Using (10) in the above equation, we get

$$S(U, V) = \frac{1}{a} \left[\left(\frac{b}{2}r - \lambda - \alpha \right) g(U, V) + \alpha\eta(U)\eta(V) \right]. \tag{14}$$

Hence we conclude the following result.

Theorem 3. *A proper Ricci-Yamabe soliton on an α -cosymplectic manifold is an η -Einstein manifold.*

If we take $\alpha = 0$, then (14) implies

$$S(U, V) = \frac{1}{a} \left(\frac{b}{2}r - \lambda \right) g(U, V).$$

Contracting the above equation, we get

$$[b(2n + 1) - 2a]r = 2\lambda(2n + 1),$$

which implies the scalar curvature is constant. Hence we have the next assertion.

Corollary 3. *If a cosymplectic manifold admits a proper Ricci-Yamabe soliton, then its scalar curvature is constant.*

Again, if $\alpha = 1$, then (14) implies

$$S(U, V) = \frac{1}{a} \left[\left(\frac{b}{2}r - \lambda - 1 \right) g(U, V) + \eta(U)\eta(V) \right].$$

Thus we have the next result.

Corollary 4. *A proper Ricci-Yamabe soliton on a Kenmotsu manifold is an η -Einstein manifold.*

3 Gradient Ricci-Yamabe solitons on 3-dimensional α -cosymplectic manifolds

We assume that an α -cosymplectic manifold admits a gradient Ricci-Yamabe soliton (g, λ, ξ, a, b) . Then from (4), we get

$$\nabla_U Df = \left(\lambda - \frac{b}{2}r \right) U - aQU. \tag{15}$$

Differentiating (15) along the vector field V , we get

$$\nabla_V \nabla_U Df = -\frac{b}{2}(Vr)U + \left(\lambda - \frac{b}{2}r \right) \nabla_V U - a\nabla_V QU. \tag{16}$$

Interchanging U and V in the above equation, we infer

$$\nabla_U \nabla_V Df = -\frac{b}{2}(Ur)V + \left(\lambda - \frac{b}{2}r\right) \nabla_U V - a \nabla_U QV. \quad (17)$$

From (15), we get

$$\nabla_{[U,V]} Df = \left(\lambda - \frac{b}{2}r\right) [U, V] - aQ([U, V]). \quad (18)$$

With the help of (16)–(18), we obtain

$$R(U, V)Df = \frac{b}{2}[(Vr)U - (Ur)V] - a[(\nabla_U Q)V - (\nabla_V Q)U].$$

Using (8) in the above equation, we get

$$\begin{aligned} R(U, V)Df &= \frac{1}{2}(b-a)[(Vr)U - (Ur)V] \\ &\quad + a\alpha \left(3\alpha^2 + \frac{r}{2}\right) [\eta(V)U - \eta(U)V] \\ &\quad + \frac{a}{2}[(Ur)\eta(V)\xi - (Vr)\eta(U)\xi]. \end{aligned} \quad (19)$$

Contracting (19), we obtain

$$S(V, Df) = \left(b - \frac{3a}{2}\right) (Vr) + a\alpha (6\alpha^2 + r) \eta(V) + \frac{a}{2}(\xi r)\eta(V). \quad (20)$$

Using (7) in (20), gives

$$S(V, Df) = \left(b - \frac{3a}{2}\right) (Vr). \quad (21)$$

Replacing U by Df in (6) and comparing with (21), we get

$$\left(\alpha^2 + \frac{r}{2}\right) (Vf) - \left(3\alpha^2 + \frac{r}{2}\right) (\xi f)\eta(V) = \frac{1}{2}(2b - 3a)(Vr). \quad (22)$$

Putting $V = \xi$ in (22) and using (7), we infer that

$$\alpha(\xi f) = \frac{1}{2}(2b - 3a) (6\alpha^2 + r). \quad (23)$$

Using (23) in (22), we get

$$\alpha \left(\alpha^2 + \frac{r}{2}\right) (Vf) - \frac{1}{2}(2b - 3a) \left[\left(3\alpha^2 + \frac{r}{2}\right) (6\alpha^2 + r) \eta(V) + \alpha(Vr)\right] = 0.$$

If we take $2b - 3a = 0$. Then the above equation implies

$$\alpha \left(\alpha^2 + \frac{r}{2}\right) (Vf) = 0.$$

The above equation implies either $\alpha = 0$ or $r = -2\alpha^2$ or $Vf = 0$.

Case I. If $\alpha = 0$, then it becomes a cosymplectic manifold.

Case II. If $r = -2\alpha^2$, then scalar curvature is a constant.

Case III. If $Vf = 0$, then f is a constant.

Thus we conclude the following result.

Theorem 4. *If a 3-dimensional α -cosymplectic manifold admits a gradient Ricci-Yamabe soliton, then either it is a cosymplectic manifold or the scalar curvature is constant or the potential function f is constant, provided $2b - 3a = 0$.*

4 Example

Consider the 5-dimensional manifold $M = \{(x_1, x_2, y_1, y_2, z) \in \mathbb{R}^5\}$, where (x_1, x_2, y_1, y_2, z) are the standard coordinates in \mathbb{R}^5 . Let e_1, e_2, e_3, e_4 and e_5 be the vector fields on M given by

$$e_1 = e^{\alpha z} \frac{\partial}{\partial x_1}, \quad e_2 = e^{\alpha z} \frac{\partial}{\partial x_2}, \quad e_3 = e^{\alpha z} \frac{\partial}{\partial y_1}, \quad e_4 = e^{\alpha z} \frac{\partial}{\partial y_2}, \quad e_5 = -\frac{\partial}{\partial z} = \zeta.$$

Let g be the Riemannian metric defined by

$$g(e_i, e_j) = \begin{cases} 0, & i \neq j, \\ 1, & i = j, \end{cases} \quad i, j = 1, 2, 3, 4, 5.$$

Let η be the 1-form on M defined by $\eta(U) = g(U, e_5) = g(U, \zeta)$ for all $U \in \chi(M)$. Let ϕ be the $(1, 1)$ tensor field on M defined by

$$\phi e_1 = -e_2, \quad \phi e_2 = e_1, \quad \phi e_3 = -e_4, \quad \phi e_4 = e_3, \quad \phi e_5 = 0.$$

By applying the linearity of ϕ and g , we have

$$\eta(\zeta) = 1, \quad \phi^2 U = -U + \eta(U)\zeta, \quad \eta(\phi U) = 0,$$

$$g(U, \zeta) = \eta(U), \quad g(\phi U, \phi V) = g(U, V) - \eta(U)\eta(V)$$

for all $U, V \in \chi(M)$. Then we have

$$[e_1, e_2] = [e_1, e_3] = [e_1, e_4] = [e_2, e_3] = [e_2, e_4] = [e_3, e_4] = 0,$$

$$[e_1, e_5] = \alpha e_1, \quad [e_2, e_5] = \alpha e_2, \quad [e_3, e_5] = \alpha e_3, \quad [e_4, e_5] = \alpha e_4.$$

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned} \nabla_{e_1} e_1 &= -\alpha e_5, & \nabla_{e_1} e_2 &= 0, & \nabla_{e_1} e_3 &= 0, & \nabla_{e_1} e_4 &= 0, & \nabla_{e_1} e_5 &= \alpha e_1, \\ \nabla_{e_2} e_1 &= 0, & \nabla_{e_2} e_2 &= -\alpha e_5, & \nabla_{e_2} e_3 &= 0, & \nabla_{e_2} e_4 &= 0, & \nabla_{e_2} e_5 &= \alpha e_2, \\ \nabla_{e_3} e_1 &= 0, & \nabla_{e_3} e_2 &= 0, & \nabla_{e_3} e_3 &= -\alpha e_5, & \nabla_{e_3} e_4 &= 0, & \nabla_{e_3} e_5 &= \alpha e_3, \\ \nabla_{e_4} e_1 &= 0, & \nabla_{e_4} e_2 &= 0, & \nabla_{e_4} e_3 &= 0, & \nabla_{e_4} e_4 &= -\alpha e_5, & \nabla_{e_4} e_5 &= \alpha e_4, \\ \nabla_{e_5} e_1 &= 0, & \nabla_{e_5} e_2 &= 0, & \nabla_{e_5} e_3 &= 0, & \nabla_{e_5} e_4 &= 0, & \nabla_{e_5} e_5 &= 0. \end{aligned}$$

It can be easily verified that the manifold satisfies

$$\nabla_U \zeta = \alpha[U - \eta(U)\zeta] \quad \text{and} \quad (\nabla_U \phi) V = \alpha[g(\phi U, V)\zeta - \eta(V)\phi U]$$

for $\zeta = e_5$. Hence the manifold is an α -cosymplectic manifold.

In [18] the authors obtained the expression of the curvature tensor and the Ricci tensor as follows

$$\begin{aligned} R(e_1, e_2)e_2 &= R(e_1, e_3)e_3 = R(e_1, e_4)e_4 = R(e_1, e_5)e_5 = -\alpha^2 e_1, \\ R(e_1, e_2)e_1 &= \alpha^2 e_2, \\ R(e_1, e_3)e_1 &= R(e_2, e_3)e_2 = R(e_5, e_2)e_5 = \alpha^2 e_3, \\ R(e_2, e_3)e_3 &= R(e_2, e_4)e_4 = R(e_2, e_5)e_5 = -\alpha^2 e_2, \\ R(e_3, e_4)e_4 &= -\alpha^2 e_3, \\ R(e_1, e_5)e_2 &= R(e_1, e_5)e_1 = R(e_4, e_5)e_4 = R(e_3, e_5)e_3 = \alpha^2 e_5, \\ R(e_1, e_4)e_1 &= R(e_2, e_4)e_2 = R(e_3, e_4)e_3 = R(e_5, e_4)e_5 = \alpha^2 e_4, \end{aligned}$$

and

$$S(e_1, e_1) = S(e_2, e_2) = S(e_3, e_3) = S(e_4, e_4) = S(e_5, e_5) = -4\alpha^2.$$

Therefore,

$$r = S(e_1, e_1) + S(e_2, e_2) + S(e_3, e_3) + S(e_4, e_4) + S(e_5, e_5) = -20\alpha^2,$$

which is a constant. From (11), we obtain

$$S(e_1, e_1) = S(e_2, e_2) = S(e_3, e_3) = S(e_4, e_4) = \frac{r}{2} - \alpha - \lambda \quad \text{and} \quad S(e_5, e_5) = \frac{r}{2} - \lambda - \mu,$$

which implies $\lambda = -6\alpha^2 - \alpha$ and $\mu = \alpha$. Therefore the data (g, ξ, λ, μ) for $\lambda = -6\alpha^2 - \alpha$ and $\mu = \alpha$ defines an η -Einstein soliton on 5-dimensional α -cosymplectic manifold and the scalar curvature is constant.

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У цій статті ми характеризуємо деякі солітони, такі як η -солітон Айнштайна, η -солітон Ямабе та солітон Річчі-Ямабе на α -косимплектичних многовидах. Крім того, ми характеризуємо 3-вимірні α -косимплектичні многовиди з градієнтним солітоном Річчі-Ямабе. Насамкінець, ми будуємо приклад.

Ключові слова і фрази: солітон Айнштайна, η -солітон Айнштайна, η -солітон Річчі, градієнтний η -солітон Річчі, η -солітон Ямабе, α -косимплектичний многовид.