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A new kind of soft algebraic structures: bipolar soft Lie algebras

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In this paper, basic concepts of soft set theory was mentioned. Then, bipolar soft Lie algebras and bipolar soft Lie ideals were defined with the help of soft sets. Some algebraic properties of the new concepts were investigated. The relationship between the two structures were analyzed. Also, it was proved that the level cuts of a bipolar soft Lie algebra were Lie subalgebras of a Lie algebra by the new definitions. After then, soft image and soft preimage of a bipolar soft Lie algebra/ideal were proved to be a bipolar soft Lie algebra/ideal.

Key words and phrases: soft set, soft Lie algebra, soft Lie ideal, bipolar soft Lie algebra, bipolar soft Lie ideal.

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Introduction

Researchers have still dealt with problems involving uncertainty for many years. It is very difficult to solve these problems by applying classical mathematical methods. Different theories have been developed to solve such problems. Some of these theories are fuzzy set theory [35], rough set theory [27] and soft set theory [25]. Fuzzy set theory has a wide range of uses in solving problems in areas such as artificial intelligence, computer science, control engineering, health sciences and natural sciences. Rough set theory has found application in solving problems in fields such as image processing, data mining, medical informatics, pattern recognition, including decision analysis, data mining, intelligent control. In recent years, soft set theory is used to solve problems in fields such as decision making, economy, engineering, medicine.

Soft set theory firstly was studied by D. Molodtsov in 1999 [25]. Also, D. Molodtsov defined soft set and operations on soft set in this study. P.K. Maji et al. investigated soft set, fuzzy soft set and intuitionisic fuzzy soft set [19–21]. In 2002, P.K. Maji et al. applied a decision making problem to soft set [22]. Thus, soft sets were used in a decision making problem for the first time. Many researchers studied this subject [7–9, 16, 18, 24, 28]. Then, H. Aktaş and N. Çağman propounded the soft group by combining groups with soft sets [2]. H. Aktaş and N. Çağman's study paved the way for many soft algebraic structures [1,3,5,6,23,26,30,32,36,37]. Also, fuzzy soft algebras were introduced in [4, 13, 17, 31, 33]. In recent years, soft set theory is widely used even in medicine. T. Herawan and M.M. Deris presented the applicability of soft set theory for decision making of patients suspected influenza [11]. S. Yüksel et al. used soft sets to diagnose the prostate cancer risk [34]. The studies increasingly continue in the medicine [10, 14, 15, 29].

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1 Preliminary

Throughout this chapter, U refers to an initial universe, E is a set of parameters, $\tilde{A} \subseteq E$ and $P(U)$ is the power set of U.

Definition 1 ([12]). *Let L be a vector space over a field H. Then, L is a Lie algebra with an operation $[\cdot,\cdot] : L \times L \longrightarrow L$ iff the following conditions are satisfied:*

1) $[a,b]$ is bilinear,

2) $[a,a] = 0$ which implies $[a,b] = -[b,a]$ for all $a,b \in L$ (skew symmetric),

3) $[[a,b],c] + [[b,c],a] + [[c,a],b] = 0$ for all $a,b,c \in L$ (Jacobi Identitiy).

Definition 2 ([12]). *Let L be a Lie algebra and $\mho : L_1 \to L_2$ be a homomorphism. Then, $\mho$ is a Lie algebra homomorphism over U iff the following condition is satisfied:*

$$\mho([a,b]) = [\mho(a), \mho(b)] \quad \text{for all} \quad a,b \in L_1.$$

Definition 3 ([25]). *A soft set $(\mathrm{Im}, \tilde{A})$ over U is a function defined by*

$$\mathrm{Im} : \tilde{A} \to P(U).$$

A soft set of E over U can be represented by the set of ordered pairs

$$(\mathrm{Im}, \tilde{A}) = \{(a, \mathrm{Im}(a)) : a \in E\}.$$

Definition 4 ([21]). *Let $(\mathrm{Im}, \tilde{A})$ be a soft set over U. If $\mathrm{Im}(a) = \varnothing$ for all $a \in E$, then $(\mathrm{Im}, \tilde{A})$ is called an empty soft set, denoted by $\widetilde{\Phi}$. If $\mathrm{Im}(a) = U$ for all $a \in E$, then $(\mathrm{Im}, \tilde{A})$ is called universal soft set, denoted by $\widetilde{A}$.*

Definition 5 ([8]). *Let $(\mathrm{Im}, \tilde{A}), (\partial, \tilde{B})$ be two softs set over U. Then, $(\mathrm{Im}, \tilde{A})$ is a soft subset of $(\partial, \tilde{B})$, denoted by $(\mathrm{Im}, \tilde{A}) \widetilde{\subseteq} (\partial, \tilde{B})$, if $\mathrm{Im}(a) \subseteq \partial(a)$ for all $a \in E$. $(\mathrm{Im}, \tilde{A})$ and $(\partial, \tilde{B})$ are equal, denoted by $(\mathrm{Im}, \tilde{A}) = (\partial, \tilde{B})$, if $\mathrm{Im}(a) = \partial(a)$ for all $a \in E$.*

Definition 6 ([8]). *Let $(\mathrm{Im}, \tilde{A}), (\partial, \tilde{B})$ be two softs set over U. Then, union $(\mathrm{Im}, \tilde{A}) \widetilde{\cup} (\partial, \tilde{B})$ and intersection $(\mathrm{Im}, \tilde{A}) \widetilde{\cap} (\partial, \tilde{B})$ of $(\mathrm{Im}, \tilde{A})$ and $(\partial, \tilde{B})$ are defined by,*

$$((\mathrm{Im} \cup \partial)(a) = \mathrm{Im}(a) \cup \partial(a), \quad (\mathrm{Im} \cap \partial)(a) = \mathrm{Im}(a) \cap \partial(a) \quad \text{for all } a \in E, \text{ respectively.}$$

Proposition 1 ([8]). *Let $(\mathrm{Im}, \tilde{A})$ be a soft set over U. Then,*

1) $(\mathrm{Im}, \tilde{A}) \widetilde{\cup} (\mathrm{Im}, \tilde{A}) = (\mathrm{Im}, \tilde{A})$, $(\mathrm{Im}, \tilde{A}) \widetilde{\cap} (\mathrm{Im}, \tilde{A}) = (\mathrm{Im}, \tilde{A})$;

2) $(\mathrm{Im}, \tilde{A}) \widetilde{\cup} \widetilde{\Phi} = (\mathrm{Im}, \tilde{A})$, $(\mathrm{Im}, \tilde{A}) \widetilde{\cap} \widetilde{\Phi} = \widetilde{\Phi}$;

3) $(\mathrm{Im}, \tilde{A}) \widetilde{\cup} \widetilde{A} = \widetilde{A}$, $(\mathrm{Im}, \tilde{A}) \widetilde{\cap} \widetilde{A} = (\mathrm{Im}, \tilde{A})$.

Proposition 2 ([8])**.** *Let* $(\mathrm{Im}, \tilde{A}), (\partial, \tilde{B})$ *and* $(\mathrm{Re}, \tilde{C})$ *be softs set over* U*. Then,*

1) $(\mathrm{Im}, \tilde{A})\tilde{\cup}(\partial, \tilde{B}) = (\partial, \tilde{B})\tilde{\cup}(\mathrm{Im}, \tilde{A})$, $(\mathrm{Im}, \tilde{A})\tilde{\cap}(\partial, \tilde{B}) = (\partial, \tilde{B})\tilde{\cap}(\mathrm{Im}, \tilde{A})$;

2) $((\mathrm{Im}, \tilde{A})\tilde{\cup}(\partial, \tilde{B}))\tilde{\cup}(\mathrm{Re}, \tilde{C}) = (\mathrm{Im}, \tilde{A})\tilde{\cup}((\partial, \tilde{B})\tilde{\cup}(\mathrm{Re}, \tilde{C}))$,
$((\mathrm{Im}, \tilde{A})\tilde{\cap}(\partial, \tilde{B}))\tilde{\cap}(\mathrm{Re}, \tilde{C}) = (\mathrm{Im}, \tilde{A})\tilde{\cap}((\partial, \tilde{B})\tilde{\cap}(\mathrm{Re}, \tilde{C}))$;

3) $(\mathrm{Im}, \tilde{A})\tilde{\cup}((\partial, \tilde{B})\tilde{\cap}(\mathrm{Re}, \tilde{C})) = ((\mathrm{Im}, \tilde{A})\tilde{\cup}(\partial, \tilde{B}))\tilde{\cap}((\mathrm{Im}, \tilde{A})\tilde{\cup}(\mathrm{Re}, \tilde{C}))$,
$(\mathrm{Im}, \tilde{A})\tilde{\cap}((\partial, \tilde{B})\tilde{\cup}(\mathrm{Re}, \tilde{C})) = ((\mathrm{Im}, \tilde{A})\tilde{\cap}(\partial, \tilde{B}))\tilde{\cup}((\mathrm{Im}, \tilde{A})\tilde{\cap}(\mathrm{Re}, \tilde{C}))$.

Definition 7 ([8])**.** *Let* $(\mathrm{Im}, \tilde{A})$ *and* $(\partial, \tilde{B})$ *be two soft sets over* U*. Then* $\curlywedge$*-product and* $\curlyvee$*-product of* $(\mathrm{Im}, \tilde{A})$ *and* $(\partial, \tilde{B})$*, denoted by* $(\mathrm{Im}, \tilde{A}) \curlywedge (\partial, \tilde{B})$ *and* $(\mathrm{Im}, \tilde{A}) \curlyvee (\partial, \tilde{B})$*, are respectively defined by*

$$(\mathrm{Im} \curlywedge \partial)(a, b) = \mathrm{Im}(a) \cap \partial(b), \quad (\mathrm{Im} \curlyvee \partial)(a, b) = \mathrm{Im}(a) \cup \partial(b) \quad \text{for all } (a, b) \in E \times E.$$

Proposition 3 ([8])**.** *Let* $(\mathrm{Im}, \tilde{A}), (\partial, \tilde{B})$ *and* $(\mathrm{Re}, \tilde{C})$ *be softs set over* U*. Then*

1) $((\mathrm{Im}, \tilde{A}) \curlywedge (\partial, \tilde{B})) \curlywedge (\mathrm{Re}, \tilde{C}) = (\mathrm{Im}, \tilde{A}) \curlywedge ((\partial, \tilde{B}) \curlywedge (\mathrm{Re}, \tilde{C}))$;

2) $((\mathrm{Im}, \tilde{A}) \curlyvee (\partial, \tilde{B})) \curlyvee (\mathrm{Re}, \tilde{C}) = (\mathrm{Im}, \tilde{A}) \curlyvee ((\partial, \tilde{B}) \curlyvee (\mathrm{Re}, \tilde{C}))$.

Definition 8 ([8])**.** *Let* $(\mathrm{Im}, \tilde{A})$ *and* $(\partial, \tilde{B})$ *be softs set over* U*. Then, the product of soft sets* $(\mathrm{Im}, \tilde{A})$ *and* $(\partial, \tilde{B})$ *is defined as* $(\mathrm{Im}, \tilde{A}) \times (\partial, \tilde{B})$*, where* $(\mathrm{Im} \times \partial)(a, b) = \mathrm{Im}(a) \times \partial(b)$ *for all* $(a, b) \in E \times E$*.*

Definition 9 ([8])**.** *Let* ∇ *be a mapping from a set* $\tilde{A}$ *to a set* $\tilde{B}$ *and let* $(\mathrm{Im}, \tilde{A})$ *be a soft set over* U*. The function*

$$\nabla(\mathrm{Im}) : \tilde{B} \to P(U), \ \nabla(\mathrm{Im})(b) = \begin{cases} \cup\{\mathrm{Im}(a) : a \in \tilde{A}, \nabla(a) = b\}, & \text{if } b \in \nabla(\tilde{A}), \\ \varnothing, & \text{if } b \notin \nabla(\tilde{A}) \end{cases}$$

for all $b \in \tilde{B}$ *is a soft set called a soft image of* $(\mathrm{Im}, \tilde{A})$ *under* ∇*. The function*

$$\nabla^{-1}(\mathrm{Im}) : \tilde{A} \to P(U), \ \nabla^{-1}(\mathrm{Im})(a) = \mathrm{Im}(\nabla(a))$$

for all $a \in \tilde{A}$ *is a soft set called a soft preimage (or soft inverse image) of* $(\mathrm{Im}, \tilde{A})$ *under* ∇*.*

Definition 10 ([8])**.** *Let* $(\mathrm{Im}, \tilde{A})$ *be a soft set over* U *and* $\tilde{T} \in P(U)$*. Then,* $\tilde{T}$*-level cut of a soft set* $(\mathrm{Im}, \tilde{A})$*, denoted by* $\mathrm{Im}_{\tilde{T}}$*, is defined as*

$$\underset{\tilde{T}}{\mathrm{Im}} = \{a \in \tilde{A} : \mathrm{Im}(a) \supseteq \tilde{T}\}.$$

2 Bipolar soft Lie algebras

Definition 11. *Let* L *be a Lie algebra. Let* (Im, L) *and* (∂, L) *be two soft sets over* U*. Then,* $(\mathrm{Im}, \partial, L)$ *is a bipolar soft Lie algebra over* U *if the following conditions are satisfied:*

1) $\mathrm{Im}(a + b) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b)$ *and* $\partial(a + b) \subseteq \partial(a) \cup \partial(b)$;

2) $\mathrm{Im}(\lambda.a) \supseteq \mathrm{Im}(a)$ *and* $\partial(\lambda.a) \subseteq \partial(a)$;

3) $\mathrm{Im}([a, b]) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b)$ *and* $\partial([a, b]) \subseteq \partial(a) \cup \partial(b)$ *for all* $a, b \in L$ *and* $\lambda \in H$*.*

Definition 12. *Let E be a Lie algebra and L be a subset of E. Let (Im, L) and (∂, L) be two soft sets over U. Then, $(\mathrm{Im}, \partial, L)$ is a bipolar soft Lie ideal over U iff the following conditions are satisfied:*

1) $\mathrm{Im}(a+b) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b)$ *and* $\partial(a+b) \subseteq \partial(a) \cup \partial(b)$*;*

2) $\mathrm{Im}(\lambda.a) \supseteq \mathrm{Im}(a)$ *and* $\partial(\lambda.a) \subseteq \partial(a)$*;*

3) $\mathrm{Im}([a,b]) \supseteq \mathrm{Im}(a)$ *and* $\partial([a,b]) \subseteq \partial(a)$ *for all* $a, b \in L$ *and* $\lambda \in H$*.*

Example 1. *Assume that $U = \mathbb{Z}$ is the universal set. Let $E = GL(2, \mathbb{R})$ of all 2×2 real matrices be a Lie algebra with $[XY] = XY - YX$.*

Let $L = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} \cup \left\{ \begin{bmatrix} 0 & 0 \\ 0 & a \end{bmatrix} : a \neq 0 \right\}$ *be a subset of E. We define soft sets (Im, L) and (∂, L) over U by*

$$\mathrm{Im}\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) = \mathbb{Z}, \qquad \mathrm{Im}\left(\begin{bmatrix} 0 & 0 \\ 0 & a \end{bmatrix}\right) = \{1,2,3,4\}$$

and

$$\partial\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) = \varnothing, \quad \partial\left(\begin{bmatrix} 0 & 0 \\ 0 & a \end{bmatrix}\right) = \{-4,-3,-2,-1\}$$

for all $a \in L$.

It can be easily shown that $(\mathrm{Im}, \partial, L)$ is a bipolar soft Lie algebra (ideal) over U.

Theorem 1. *Let $(\mathrm{Im}, \partial, L)$ be a bipolar soft Lie algebra/ideal over U. Then,*

1) $\mathrm{Im}(0) \supseteq \mathrm{Im}(a)$ *and* $\partial(0) \subseteq \partial(a)$*;*

2) $\mathrm{Im}(-a) \supseteq \mathrm{Im}(a)$ *and* $\partial(-a) \subseteq \partial(a)$ *for all* $a \in L$*.*

Proof. For all $a \in L$, it follows that:

1) $\mathrm{Im}(0) = \mathrm{Im}(0.a) \supseteq \mathrm{Im}(a)$ and $\partial(0) = \partial(0.a) \subseteq \partial(a)$;

2) $\mathrm{Im}(-a) = \mathrm{Im}(-1.a) \supseteq \mathrm{Im}(a)$ and $\partial(-a) = \partial(-1.a) \subseteq \partial(a)$. □

Theorem 2. *Let $(\mathrm{Im}_1, \partial_1, L)$ and $(\mathrm{Im}_2, \partial_2, L)$ be two bipolar soft Lie algebras/ideals over U. Then, $(\mathrm{Im}_1 \curlywedge \mathrm{Im}_2, \partial_1 \curlyvee \partial_2, L \times L)$ is a bipolar soft Lie algebra/ideal over $U \times U$.*

Proof. For all $a, b, c, d \in L$ and $\lambda \in H$, we have the following.

1)

$$\begin{aligned}(\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)((a,b)+(c,d)) &= (\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)(a+c, b+d) = \mathrm{Im}_1(a+c) \cap \mathrm{Im}_2(b+d) \\ &\supseteq \mathrm{Im}_1(a) \cap \mathrm{Im}_1(c) \cap \mathrm{Im}_2(b) \cap \mathrm{Im}_2(d) = \mathrm{Im}_1(a) \cap \mathrm{Im}_2(b) \cap \mathrm{Im}_1(c) \cap \mathrm{Im}_2(d) \\ &= (\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)(a,b) \cap (\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)(c,d).\end{aligned}$$

Then, $(\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)((a,b)+(c,d)) \supseteq (\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)(a,b) \cap (\mathrm{Im}_1 \curlywedge \mathrm{Im}_2)(c,d)$ is obtained.

$$\begin{aligned}(\partial_1 \curlyvee \partial_2)((a,b)+(c,d)) &= (\partial_1 \curlyvee \partial_2)(a+c, b+d) = \partial_1(a+c) \cup \partial_2(b+d) \\ &\subseteq \partial_1(a) \cup \partial_1(c) \cup \partial_2(b) \cup \partial_2(d) = \partial_1(a) \cup \partial_2(b) \cup \partial_1(c) \cup \partial_2(d) \\ &= (\partial_1 \curlyvee \partial_2)(a,b) \cup (\partial_1 \curlyvee \partial_2)(c,d).\end{aligned}$$

Then, $(\partial_1 \curlyvee \partial_2)((a,b)+(c,d)) \subseteq (\partial_1 \curlyvee \partial_2)(a,b) \cup (\partial_1 \curlyvee \partial_2)(c,d)$ is obtained.

2)

$$(\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})(\lambda.(a,b)) = (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})(\lambda.a, \lambda.b) = \underset{1}{\text{Im}}(\lambda.a) \cap \underset{2}{\text{Im}}(\lambda.b)$$
$$\supseteq \underset{1}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(b) = \underset{1}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(b) \cap \underset{1}{\text{Im}}(c) \cap \underset{2}{\text{Im}}(d) = (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})(a,b).$$

Then, $(\text{Im}_1 \curlywedge \text{Im}_2)(\lambda.(a,b)) \supseteq (\text{Im}_1 \curlywedge \text{Im}_2)(a,b)$ is obtained.

Similarly, it follows that $(\partial_1 \curlyvee \partial_2)(\lambda.(a,b)) \subseteq (\partial_1 \curlyvee \partial_2)(a,b)$.

3)

$$(\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})([(a,b),(c,d)]) = (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})([a,c],[b,d]) = \underset{1}{\text{Im}}([a,c]) \cap \underset{2}{\text{Im}}([b,d])$$
$$\supseteq \underset{1}{\text{Im}}(a) \cap \underset{1}{\text{Im}}(c) \cap \underset{2}{\text{Im}}(b) \cap \underset{2}{\text{Im}}(d) = \underset{1}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(b) \cap \underset{1}{\text{Im}}(c) \cap \underset{2}{\text{Im}}(d)$$
$$= (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})(a,b) \cap (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})(c,d).$$

Then, $(\text{Im}_1 \curlywedge \text{Im}_2)([(a,b),(c,d)]) \supseteq (\text{Im}_1 \curlywedge \text{Im}_2)(a,b) \cap (\text{Im}_1 \curlywedge \text{Im}_2)(c,d)$ is obtained.

Similarly, it follows that $(\partial_1 \curlyvee \partial_2)([(a,b),(c,d)]) \subseteq (\partial_1 \curlyvee \partial_2)(a,b) \cup (\partial_1 \curlyvee \partial_2)(c,d)$. Thus, $(\text{Im}_1 \curlywedge \text{Im}_2, \partial_1 \curlyvee \partial_2, L \times L)$ is a bipolar soft Lie algebra over $U \times U$.

4)

$$(\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})([(a,b),(c,d)]) = (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})([a,c],[b,d])$$
$$= \underset{1}{\text{Im}}([a,c]) \cap \underset{2}{\text{Im}}([b,d]) \supseteq \underset{1}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(b) = (\underset{1}{\text{Im}} \curlywedge \underset{2}{\text{Im}})(a,b).$$

Then, $(\text{Im}_1 \curlywedge \text{Im}_2)([(a,b),(c,d)]) \supseteq (\text{Im}_1 \curlywedge \text{Im}_2)(a,b)$ is obtained. Similarly, it follows that $(\partial_1 \curlyvee \partial_2)([(a,b),(c,d)]) \subseteq (\partial_1 \curlyvee \partial_2)(a,b)$. Thus, $(\text{Im}_1 \curlywedge \text{Im}_2, \partial_1 \curlyvee \partial_2, L \times L)$ is a bipolar soft Lie ideal over $U \times U$. □

Theorem 3. *Let* $(\text{Im}_1, \partial_1, L)$ *and* $(\text{Im}_2, \partial_2, L)$ *be two bipolar soft Lie algebras/ideals over* U. *Then,* $(\text{Im}_1 \tilde{\cap} \text{Im}_2, \partial_1 \tilde{\cup} \partial_2, L)$ *is a bipolar soft Lie algebra/ideal over* U.

Proof. For all $a, b \in L$ and $\lambda \in H$, we have the following.

1)

$$(\underset{1}{\text{Im}} \cap \underset{2}{\text{Im}})((a+b) = \underset{1}{\text{Im}}(a+b) \cap \underset{2}{\text{Im}}(a+b) \supseteq \underset{1}{\text{Im}}(a) \cap \underset{1}{\text{Im}}(b) \cap \underset{2}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(b)$$
$$= \underset{1}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(a) \cap \underset{1}{\text{Im}}(b) \cap \underset{2}{\text{Im}}(b) = (\underset{1}{\text{Im}} \cap \underset{2}{\text{Im}})(a) \cap (\underset{1}{\text{Im}} \cap \underset{2}{\text{Im}})(b).$$

Then, $(\text{Im}_1 \cap \text{Im}_2)(a+b) \supseteq (\text{Im}_1 \cap \text{Im}_2)(a) \cap (\text{Im}_1 \cap \text{Im}_2)(b)$ is obtained.

$$(\partial_1 \cup \partial_2)(a+b) = \partial_1(a+b) \cup \partial_2(a+b) \subseteq \partial_1(a) \cup \partial_1(b) \cup \partial_2(a) \cup \partial_2(b)$$
$$= \partial_1(a) \cup \partial_2(a) \cup \partial_1(b) \cup \partial_2(b) = (\partial_1 \cup \partial_2)(a) \cup (\partial_1 \cup \partial_2)(b).$$

Then, $(\partial_1 \cup \partial_2)(a+b) \subseteq (\partial_1 \cup \partial_2)(a) \cup (\partial_1 \cup \partial_2)(b)$ is obtained.

2)

$$(\underset{1}{\text{Im}} \cap \underset{2}{\text{Im}})(\lambda.a) = \underset{1}{\text{Im}}(\lambda.a) \cap \underset{2}{\text{Im}}(\lambda.a) \supseteq \underset{1}{\text{Im}}(a) \cap \underset{2}{\text{Im}}(a) = (\underset{1}{\text{Im}} \cap \underset{2}{\text{Im}})(a).$$

Then, $(\text{Im}_1 \cap \text{Im}_2)(\lambda.a) \supseteq (\text{Im}_1 \cap \text{Im}_2)(a)$ is obtained.

Similarly, it follows that $(\partial_1 \cup \partial_2)(\lambda.a) \subseteq (\partial_1 \cup \partial_2)(a)$.

3)

$$\begin{aligned}(\underset{1}{\mathrm{Im}}\cap\underset{2}{\mathrm{Im}})([a,b]) &= \underset{1}{\mathrm{Im}}([a,b])\cap\underset{2}{\mathrm{Im}}([a,b]) \supseteq \underset{1}{\mathrm{Im}}(a)\cap\underset{1}{\mathrm{Im}}(b)\cap\underset{2}{\mathrm{Im}}(a)\cap\underset{2}{\mathrm{Im}}(b)\\ &= \underset{1}{\mathrm{Im}}(a)\cap\underset{2}{\mathrm{Im}}(a)\cap\underset{1}{\mathrm{Im}}(b)\cap\underset{2}{\mathrm{Im}}(b) = (\underset{1}{\mathrm{Im}}\cap\underset{2}{\mathrm{Im}})(a)\cap(\underset{1}{\mathrm{Im}}\cap\underset{2}{\mathrm{Im}})(b).\end{aligned}$$

Then, $(\mathrm{Im}_1\cap\mathrm{Im}_2)([a,b]) \supseteq (\mathrm{Im}_1\cap\mathrm{Im}_2)(a)\cap(\mathrm{Im}_1\cap\mathrm{Im}_2)(b)$ is obtained. Similarly, it follows that $(\partial_1\cup\partial_2)([a,b]) \subseteq (\partial_1\cup\partial_2)(a)\cup(\partial_1\cup\partial_2)(b)$. Thus, $(\mathrm{Im}_1\cap\mathrm{Im}_2,\partial_1\cup\partial_2,L)$ is a bipolar soft Lie algebra over U.

4)

$$(\underset{1}{\mathrm{Im}}\cap\underset{2}{\mathrm{Im}})([a,b]) = \underset{1}{\mathrm{Im}}([a,b])\cap\underset{2}{\mathrm{Im}}([a,b] \supseteq \underset{1}{\mathrm{Im}}(a)\cap\underset{2}{\mathrm{Im}}(a) = (\underset{1}{\mathrm{Im}}\cap\underset{2}{\mathrm{Im}})(a).$$

Then, $(\mathrm{Im}_1\cap\mathrm{Im}_2)([a,b]) \supseteq (\mathrm{Im}_1\cap\mathrm{Im}_2)(a)$ is obtained.

Similarly, it follows that $(\partial_1\cup\partial_2)([a,b]) \subseteq (\partial_1\cup\partial_2)(a)$. Thus, $(\mathrm{Im}_1\cap\mathrm{Im}_2,\partial_1\cup\partial_2,L)$ is a bipolar soft Lie ideal over U. □

Theorem 4. *Let* $(\mathrm{Im}_1,\partial_1,L)$ *and* $(\mathrm{Im}_2,\partial_2,L)$ *be two bipolar soft Lie algebras/ideals over* U. *Then,* $(\mathrm{Im}_1\times\mathrm{Im}_2,\partial_1\times\partial_2,L\times L)$ *is a bipolar soft Lie algebra/ideal over* $U\times U$.

Proof. For all $a,b,c,d\in L$ and $\lambda\in H$, we have the following.

1)

$$\begin{aligned}(\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})((a,b)+(c,d)) &= (\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(a+c,b+d) = \underset{1}{\mathrm{Im}}(a+c)\times\underset{2}{\mathrm{Im}}(b+d)\\ &\supseteq [\underset{1}{\mathrm{Im}}(a)\cap\underset{1}{\mathrm{Im}}(c)]\times[\underset{2}{\mathrm{Im}}(b)\cap\underset{2}{\mathrm{Im}}(d)]\\ &= [\underset{1}{\mathrm{Im}}(a)\times\underset{2}{\mathrm{Im}}(b)]\cap[\underset{1}{\mathrm{Im}}(c)\times\underset{2}{\mathrm{Im}}(d)]\\ &= (\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(a,b)\cap(\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(c,d).\end{aligned}$$

Then, $(\mathrm{Im}_1\times\mathrm{Im}_2)((a,b)+(c,d)) \supseteq (\mathrm{Im}_1\times\mathrm{Im}_2)(a,b)\cap(\mathrm{Im}_1\times\mathrm{Im}_2)(c,d)$ is obtained.

$$\begin{aligned}(\partial_1\times\partial_2)((a,b)+(c,d)) &= (\partial_1\times\partial_2)(a+c,b+d) = \partial_1(a+c)\times\partial_2(b+d)\\ &\subseteq [\partial_1(a)\cup\partial_1(c)]\times[\partial_2(b)\cup\partial_2(d)] = [\partial_1(a)\times\partial_2(b)]\cup[\partial_1(c)\times\partial_2(d)]\\ &= (\partial_1\times\partial_2)(a,b)\cup(\partial_1\times\partial_2)(c,d).\end{aligned}$$

Then, $(\partial_1\times\partial_2)((a,b)+(c,d)) \subseteq (\partial_1\times\partial_2)(a,b)\cup(\partial_1\times\partial_2)(c,d)$ is obtained.

2)

$$\begin{aligned}(\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(\lambda.(a,b)) &= (\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(\lambda.a,\lambda.b) = \underset{1}{\mathrm{Im}}(\lambda.a)\times\underset{2}{\mathrm{Im}}(\lambda.b)\\ &\supseteq \underset{1}{\mathrm{Im}}(a)\times\underset{2}{\mathrm{Im}}(b) = (\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(a,b).\end{aligned}$$

Then, $(\mathrm{Im}_1\times\mathrm{Im}_2)(\lambda.(a,b)) \supseteq (\mathrm{Im}_1\times\mathrm{Im}_2)(a,b)$ is obtained.

Similarly, it follows that $(\partial_1\times\partial_2)(\lambda.(a,b)) \subseteq (\partial_1\times\partial_2)(a,b)$.

3)

$$\begin{aligned}(\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})([(a,b),(c,d)]) &= (\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})([a,c],[b,d]) = \underset{1}{\mathrm{Im}}([a,c])\times\underset{2}{\mathrm{Im}}([b,d])\\ &\supseteq [\underset{1}{\mathrm{Im}}(a)\cap\underset{1}{\mathrm{Im}}(c)]\times[\underset{2}{\mathrm{Im}}(b)\cap\underset{2}{\mathrm{Im}}(d)]\\ &= [\underset{1}{\mathrm{Im}}(a)\times\underset{2}{\mathrm{Im}}(b)]\cap[\underset{1}{\mathrm{Im}}(c)\times\underset{2}{\mathrm{Im}}(d)]\\ &= (\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(a,b)\cap(\underset{1}{\mathrm{Im}}\times\underset{2}{\mathrm{Im}})(c,d).\end{aligned}$$

Then, $(\mathrm{Im}_1 \times \mathrm{Im}_2)([(a,b),(c,d)]) \supseteq (\mathrm{Im}_1 \times \mathrm{Im}_2)(a,b) \cap (\mathrm{Im}_1 \times \mathrm{Im}_2)(c,d)$ is obtained.

Similarly, it follows that $(\partial_1 \times \partial_2)([(a,b),(c,d)]) \subseteq (\partial_1 \times \partial_2)(a,b) \cup (\partial_1 \times \partial_2)(c,d)$. Thus, $(\mathrm{Im}_1 \times \mathrm{Im}_2, \partial_1 \times \partial_2, L \times L)$ is a bipolar soft Lie algebra over $U \times U$.

4)

$$\begin{aligned}(\underset{1}{\mathrm{Im}} \times \underset{2}{\mathrm{Im}})([(a,b),(c,d)]) &= (\underset{1}{\mathrm{Im}} \times \underset{2}{\mathrm{Im}})([a,c],[b,d]) \\ &= \underset{1}{\mathrm{Im}}([a,c]) \times \underset{2}{\mathrm{Im}}([b,d]) \supseteq \underset{1}{\mathrm{Im}}(a) \times \underset{2}{\mathrm{Im}}(b) = (\underset{1}{\mathrm{Im}} \times \underset{2}{\mathrm{Im}})(a,b).\end{aligned}$$

Then, $(\mathrm{Im}_1 \times \mathrm{Im}_2)([(a,b),(c,d)]) \supseteq (\mathrm{Im}_1 \times \mathrm{Im}_2)(a,b)$ is obtained. Similarly, it follows that $(\partial_1 \times \partial_2)([(a,b),(c,d)]) \subseteq (\partial_1 \times \partial_2)(a,b)$. Thus, $(\mathrm{Im}_1 \times \mathrm{Im}_2, \partial_1 \times \partial_2, L \times L)$ is a bipolar soft Lie ideal over $U \times U$. □

Theorem 5. *Every bipolar soft Lie ideal is a bipolar soft Lie algebra.*

Proof. The proof can be easily obtained from Definition 11 and Definition 12. □

Theorem 6. *Let $(\mathrm{Im}, \partial, L)$ be a bipolar soft set over U. Then, $(\mathrm{Im}, \partial, L)$ is a bipolar soft Lie algebra over U if and only if the non-empty upper $\tilde{K}$-level cut $\mathrm{Im}_{\tilde{K}}^{\supseteq} = \{a \in L : \mathrm{Im}(a) \supseteq \tilde{K}\}$ and the non-empty lower $\tilde{T}$-level cut $\partial_{\tilde{T}}^{\subseteq} = \{a \in L : \partial(a) \subseteq \tilde{T}\}$ are Lie subalgebras of L for all $\tilde{K}, \tilde{T} \in P(U)$.*

Proof. Suppose that $(\mathrm{Im}, \partial, L)$ is a bipolar soft Lie algebra over U. Let $a, b \in \mathrm{Im}_{\tilde{K}}^{\supseteq}$. Then, $\mathrm{Im}(a) \supseteq \tilde{K}$ and $\mathrm{Im}(b) \supseteq \tilde{K}$. It follows that

$$\mathrm{Im}(a+b) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b) \supseteq \tilde{K} \cap \tilde{K} = \tilde{K}, \qquad \mathrm{Im}(\lambda.a) \supseteq \mathrm{Im}(a) \supseteq \tilde{K},$$

$$\mathrm{Im}([a,b]) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b) \supseteq \tilde{K} \cap \tilde{K} = \tilde{K}$$

and thus, $a+b \in \mathrm{Im}_{\tilde{K}}^{\supseteq}$, $\lambda.a \in \mathrm{Im}_{\tilde{K}}^{\supseteq}$, $[a,b] \in \mathrm{Im}_{\tilde{K}}^{\supseteq}$. Therefore, $\mathrm{Im}_{\tilde{K}}^{\supseteq}$ is a Lie subalgebra of L for all $\tilde{K} \in P(U)$. Similarly, $\partial_{\tilde{T}}^{\subseteq}$ is a Lie subalgebra of L for all $\tilde{T} \in P(U)$.

Conversely, suppose that $\mathrm{Im}_{\tilde{K}}^{\supseteq}$ and $\partial_{\tilde{T}}^{\subseteq}$ are Lie subalgebras of L for all $\tilde{K}, \tilde{L} \in P(U)$. Let $\mathrm{Im}(a) = \tilde{A}$, $\mathrm{Im}(b) = \tilde{B}$ be such that $\tilde{A} \subseteq \tilde{B}$ for all $\tilde{A}, \tilde{B} \in P(U)$ and $a, b \in L$. It follows that $a, b \in \mathrm{Im}_{\tilde{A}}^{\supseteq}$. Since $\mathrm{Im}_{\tilde{A}}^{\supseteq}$ is a Lie subalgebra of L, then $a + b \in \mathrm{Im}_{\tilde{A}}^{\supseteq}$. Thus, $\mathrm{Im}(a+b) \supseteq \tilde{A} = \tilde{A} \cap \tilde{B} = \mathrm{Im}(a) \cap \mathrm{Im}(b)$. Therefore, $\mathrm{Im}(a+b) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b)$. Since $a \in \mathrm{Im}_{\tilde{A}}^{\supseteq}$ and $\mathrm{Im}_{\tilde{A}}^{\supseteq}$ is a Lie subalgebra of L, then $\lambda.a \in \mathrm{Im}_{\tilde{A}}^{\supseteq}$ for all $\lambda \in H$. Thus, $\mathrm{Im}(\lambda.a) \supseteq \tilde{A} = \mathrm{Im}(a)$. Therefore, $\mathrm{Im}(\lambda.a) \supseteq \mathrm{Im}(a)$. Since $a, b \in \mathrm{Im}_{\tilde{A}}^{\supseteq}$ and $\mathrm{Im}_{\tilde{A}}^{\supseteq}$ is a Lie subalgebra of L, then $[a,b] \in \mathrm{Im}_{\tilde{A}}^{\supseteq}$. Thus, $\mathrm{Im}([a,b]) \supseteq \tilde{A} = \tilde{A} \cap \tilde{B} = \mathrm{Im}(a) \cap \mathrm{Im}(b)$. Therefore, $\mathrm{Im}([a,b]) \supseteq \mathrm{Im}(a) \cap \mathrm{Im}(b)$. Similarly, it follows that $\partial(a+b) \subseteq \partial(a) \cup \partial(b)$, $\partial(\lambda.a) \subseteq \partial(a)$ and $\partial([a,b]) \subseteq \partial(a) \cup \partial(b)$ for all $a, b \in L$ and $\lambda \in H$. Hence, $(\mathrm{Im}, \partial, L)$ is a bipolar soft Lie algebra over U. □

Theorem 7. *Let L_1 and L_2 be two Lie algebras. $(\mathrm{Im}, \partial, L_1)$ a bipolar soft Lie algebras/ideals over U and $\mho : L_1 \to L_2$ be a surjective Lie homomorphism. Then, $(\mho(\mathrm{Im}), \mho(\partial), L_2)$ is a bipolar soft Lie algebra/ideal over U.*

Proof. We know that $\mho$ is an isomorphism from L_1 to L_2. Therefore, there exist $a, b \in L_1$ such that $x = \mho(a)$ and $y = \mho(b)$ for all $x, y \in L_2$. For all $a, b \in L_1$ and $\lambda \in H$, we have the following.

1)

$$\begin{aligned}\mho(\text{Im})((x+y) &= \cup\{\text{Im}(z) : z \in L_1, \mho(z) = x+y\}\\ &= \cup\{\text{Im}(a+b) : x, y \in L_2, \mho(a) = x, \mho(b) = y\}\\ &\supseteq \cup\{\text{Im}(a) \cap \text{Im}(b) : a, b \in L_1, \mho(a) = x, \mho(b) = y\}\\ &= (\cup\{\text{Im}(a) : a \in L_1, \mho(a) = x\}) \cap (\cup\{\text{Im}(b) : b \in L_1, \mho(b) = y\})\\ &= (\mho(\text{Im}))(x) \cap (\mho(\text{Im}))(y).\end{aligned}$$

Then, $\mho(\text{Im})(x+y) \supseteq \mho(\text{Im})(x) \cap \mho(\text{Im})(y)$ is obtained.

$$\begin{aligned}\mho(\partial)(x+y) &= \cap\{\partial(z) : z \in L_1, \mho(z) = x+y\} = \cap\{\partial(a+b) : x, y \in L_2, \mho(a) = x, \mho(b) = y\}\\ &\subseteq \cap\{\partial(a) \cup \partial(b) : a, b \in L_1, \mho(a) = x, \mho(b) = y\}\\ &= (\cap\{\partial(a) : a \in L_1, \mho(a) = x\}) \cup (\cap\{\partial(b) : b \in L_1, \mho(b) = y\})\\ &= (\mho(\partial))(x) \cup (\mho(\partial))(y).\end{aligned}$$

Then, $\mho(\partial)(x+y) \subseteq \mho(\partial)(x) \cup \mho(\partial)(y)$ is obtained.
2)

$$\begin{aligned}\mho(\text{Im})(\lambda.x) &= \cup\{\text{Im}(z) : z \in L_1, \mho(z) = \lambda.x\} = \cup\{\text{Im}(\lambda.a) : x \in L_2, \mho(a) = x\}\\ &\supseteq \cup\{\text{Im}(a) : a \in L_1, \mho(a) = x\} = \mho(\text{Im})(x).\end{aligned}$$

Then, $\mho(\text{Im})(\lambda.x) \supseteq \mho(\text{Im})(x)$ is obtained. Similarly, it follows that $\mho(\partial)(\lambda.x) \subseteq \mho(\partial)(x)$.
3)

$$\begin{aligned}\mho(\text{Im})([x,y]) &= \cup\{\text{Im}(z) : z \in L_1, \mho(z) = [x,y]\}\\ &= \cup\{\text{Im}([a,b]) : x, y \in L_2, \mho(a) = x, \mho(b) = y\}\\ &\supseteq \cup\{\text{Im}(a) \cap \text{Im}(b) : a, b \in L_1, \mho(a) = x, \mho(b) = y\}\\ &= (\cup\{\text{Im}(a) : a \in L_1, \mho(a) = x\}) \cap (\cup\{\text{Im}(b) : b \in L_1, \mho(b) = y\})\\ &= (\mho(\text{Im}))(x) \cap (\mho(\text{Im}))(y).\end{aligned}$$

Then, $\mho(\text{Im})([x,y]) \supseteq \mho(\text{Im})(x) \cap \mho(\text{Im})(y)$ is obtained.

Similarly, it follows that $\mho(\partial)([x,y]) \subseteq \mho(\partial)(x) \cup \mho(\partial)(y)$. Thus, $(\mho(\text{Im}), \mho(\partial), L_2)$ is a bipolar soft Lie algebra over U.
4)

$$\begin{aligned}\mho(\text{Im})([x,y]) &= \cup\{\text{Im}(z) : z \in L_1, \mho(z) = [x,y]\}\\ &= \cup\{\text{Im}([a,b]) : x, y \in L_2, \mho(a) = x, \mho(b) = y\}\\ &\supseteq \cup\{\text{Im}(a) : a \in L_1, \mho(a) = x\}\\ &= (\cup\{\text{Im}(a) : a \in L_1, \mho(a) = x\}) = (\mho(\text{Im}))(x).\end{aligned}$$

Then, $\mho(\text{Im})([x,y]) \supseteq \mho(\text{Im})(x)$ is obtained.

Similarly, it follows that $\mho(\partial)([x,y]) \subseteq \mho(\partial)(x)$. Thus, $(\mho(\text{Im}), \mho(\partial), L_2)$ is a bipolar soft Lie ideal over U. □

Theorem 8. *Let L_1 and L_2 be two Lie algebras.* $(\mathrm{Im}, \partial, L_2)$ *a bipolar soft Lie algebras/ideals over U and $\mho : L_1 \to L_2$ be a Lie homomorphism. Then,* $(\mho^{-1}(\mathrm{Im}), \mho^{-1}(\partial), L_1)$ *is a bipolar soft Lie algebra/ideal over U.*

Proof. For all $a, b \in L_1$ and $\lambda \in H$, we have the following.
1)

$$\begin{aligned}\mho^{-1}(\mathrm{Im})(a+b) = \mathrm{Im}(\mho(a+b)) &= \mathrm{Im}(\mho(a)+\mho(b)) \\ &\supseteq \mathrm{Im}(\mho(a)) \cap \mathrm{Im}(\mho(b)) = \mho^{-1}\,\mathrm{Im}(a) \cap \mho^{-1}\,\mathrm{Im}(b).\end{aligned}$$

Then, $\mho^{-1}(\mathrm{Im})(a+b) \supseteq \mho^{-1}(\mathrm{Im})(a) \cap \mho(\mathrm{Im})(b)$ is obtained.

$$\mho^{-1}(\partial)(a+b) = \partial(\mho(a+b)) = \partial(\mho(a)+\mho(b)) \subseteq \partial(\mho(a)) \cup \partial(\mho(b)) = \mho^{-1}\partial(a) \cap \mho^{-1}\partial(b).$$

Then, $\mho^{-1}(\partial)(a+b) \subseteq \mho^{-1}(\partial)(a) \cup \mho(\partial)(b)$ is obtained.
2)

$$\mho^{-1}(\mathrm{Im})(\lambda.a) = \mathrm{Im}(\mho(\lambda.a)) = \mathrm{Im}(\lambda.\mho(a)) \supseteq \mathrm{Im}(\mho(a)).$$

Then, $\mho^{-1}(\mathrm{Im})(\lambda.a) \supseteq \mho^{-1}(\mathrm{Im})(a)$ is obtained.

Similarly, it follows that $\mho^{-1}(\partial)(\lambda.a) \subseteq \mho^{-1}(\partial)(a)$.
3)

$$\mho^{-1}(\mathrm{Im})([a,b]) = \mathrm{Im}(\mho([a,b])) \supseteq \mathrm{Im}(\mho(a)) \cap \mathrm{Im}(\mho(b)).$$

Then, $\mho^{-1}(\mathrm{Im})([a,b]) \supseteq \mho^{-1}(\mathrm{Im})(a) \cap \mho^{-1}(\mathrm{Im})(b)$ is obtained. Similarly, it follows that $\mho^{-1}(\partial)([a,b]) \subseteq \mho^{-1}(\partial)(a) \cup \mho^{-1}(\partial)(b)$. Thus, $(\mho^{-1}(\mathrm{Im}), \mho^{-1}(\partial), L_1)$ is a bipolar soft Lie algebra over U.
4)

$$\mho^{-1}(\mathrm{Im})([a,b]) = \mathrm{Im}(\mho([a,b])) \supseteq \mathrm{Im}(\mho(a)).$$

Then, $\mho^{-1}(\mathrm{Im})([a,b]) \supseteq \mho^{-1}(\mathrm{Im})(a)$ is obtained.

Similarly, it follows that $\mho^{-1}(\partial)([a,b]) \subseteq \mho^{-1}(\partial)(a)$. Thus, $(\mho^{-1}(\mathrm{Im}), \mho^{-1}(\partial), L_1)$ is a bipolar soft Lie ideal over U. □

3 Conclusion

In this paper, we defined bipolar soft Lie algebras and bipolar soft Lie ideals with the help of soft sets. We investigated some algebraic properties of the new concepts. We analyzed the relationship between the two structures. Also it was proved that the level cuts of a bipolar soft Lie algebra were Lie subalgebras of a Lie algebra by the new definitions. After then, it was proved that the soft image and the soft preimage of a bipolar soft Lie algebra/ideal were a bipolar soft Lie algebra/ideal. Based on this study, researcher can define bipolar soft WS-algebras. And, the properties of bipolar soft WS-algebras investigated.

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Чітак Ф. *Новий тип м'яких алгебраїчних структур: біполярні м'які алгебри Лі* // Карпатські матем. публ. — 2022. — Т.14, №2. — C. 464–474.

У цій статті ми описуємо базові концепції теорії м'яких множин. Ми означаємо біполярні м'які алгебри Лі та біполярні м'які ідеали Лі за допомогою м'яких множин. Ми дослідили деякі алгебраїчні властивості цих нових структур та проаналізували взаємозв'язок між ними. Також доведено, що рівневі розрізи біполярної м'якої алгебри Лі є підалгебрами Лі алгебри Лі в сенсі нових означень. Насамкінець доведено, що м'який образ та м'який прообраз біполярної м'якої алгебри Лі (біполярного м'якого ідеалу Лі) є біполярною м'якою алгеброю Лі (біполярний м'яким ідеалом Лі).

Ключові слова і фрази: м'яка множина, м'яка алгебра Лі, м'який ідеал Лі, біполярна м'яка алгебра Лі, біполярний м'який ідеал Лі.