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GROUPS ASSOCIATED WITH BRACES

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We construct the group $H(A)$ associated with a brace A and investigate the properties of $H(A)$.

Introduction

Let $(A, +)$ be an abelian group with a multiplication "$\cdot$". As in [5] we call A a brace if A is right distributive, i.e.

i) $(a+b)\cdot c{=}(a\cdot c)+(b\cdot c)$ for all $a, b, c \in A$, and

ii) A is a group with respect to circle operation "$\circ$" defined by the rule

$$a \circ b = a + b + a \cdot b.$$

A group $(A, \circ)$ is called the adjoint group of a brace A and denoted by A°. It is easy to see that

$$a \circ 0 = 0 = 0 \circ a$$

and so 0 is the neutral element of A°. The inverse of $a \in A$ will be denoted by $a^{(-1)}$.

An abelian group $(M, +)$ is called a module [6] (with the neutral element e) over a brace A if there exists a mapping

$$M \times A \ni (x, a) \mapsto xa \in M$$

such that the following hold for any elements $x, y \in M$ and $a, b \in A$:

$m_1)$ $(x + y)a = xa + ya$,

$m_2)$ $x(a \circ b) = (xa)b + xa + xb$,

$m_3)$ $x0 = e$.

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Since

$$ea = (e + e)a = ea + ea,$$

we conclude that

$$ea = e$$

for any $a \in A$. In view of $x + (-x) = e$ we also obtain that

$$0 = ea = (x + (-x))a = xa + (-x)a$$

and therefore

$$(-x)a = -(xa) = -xa.$$

A non-empty set $L \subseteq M$ is called a submodule of a module M if the following two conditions hold:

s_1) L is a subgroup of $(M, +)$,

s_2) $la \in L$ for any $l \in L$ and $a \in A$.

Let A be a brace, L a submodule of an A-module M, T a subgroup of A°. On the set of pairs

$$H(L, T) = \{(l, t) \mid l \in L, t \in T\}$$

we define a multiplication by the rule

$$(x, y)(u, v) = (xv + x + u, y \circ v) \tag{1}$$

for $x, u \in L$ and $y, v \in T$. Then $H(L, T)$ is a group (see Lemma 1). We prove the following

Theorem 1. *Let M be a module over a brace A, L a non-zero submodule of M, T a non-zero subgroup of A°. Then*

$$H = H(L, T) = E \rtimes F$$

is a Frobenius group with a kernel E and a complement F, where E is isomorphic to the additive group L^+ of L and F is isomorphic to a subgroup T, if and only if the following hold:

(i) $L = Lh$ *for every non-zero element* $h \in T$,

(ii) $\operatorname{ann}_T l = \{t \in T \mid lt = e\} = \{0\}$ *for every non-zero element* $l \in L$.

Recall [5] that

$$A^{n+1} = A(A^n)$$

and

$$A^{(n+1)} = (A^{(n)})A$$

for any positive integer n. Then A^n is a right ideal and $A^{(n)}$ is a two-sided ideal in A. A brace A is called right nilpotent (respectively left nilpotent) if $A^{(n)} = \{0\}$ for some positive integer n. A minimal positive integer n with this property is called an index of right (respectively left) nilpotency. In this way we obtain the following

Theorem 2. (1) *If A is a non-zero left nilpotent brace, then*

- *(i)* $H(A)$ *is a nilpotent group;*
- *(ii)* $\operatorname{ann} A \neq \{0\}$.

(2) *If A is a right nilpotent brace, then $H(A)$ is a solvable group.*

Henceforth, $H \lhd G$ means that H is a normal subgroup of a group G and $E \rtimes F$ is a semidirect product of groups E, F with a normal subgroup E.

Any unexplaned terminology is standard as in [4].

1. The group associated with a brace.

It is not difficult to prove the following

Lemma 1. *Let M be a module over a brace A. If L is a submodule of M and T is a subgroup of A°, then*

$$H = H(L, T) = E \rtimes F$$

is a group with the identity element $(e, 0)$ under the operation (1) and, moreover, $E = \{(l, 0) \mid l \in L\}$ is isomorphic to the additive group of L and $F = \{(e, t) \mid t \in T\}$ is isomorphic to T.

Proof. It is easily verified that $H(L, T)$ is a group, for any $a, l \in L$, $b \in T$

$$(a, b)^{-1} = (-a - ab^{(-1)}, b^{(-1)}) \in H$$

and

$$(a, b) = (e, b)(a, 0) \in EF,$$

$$(l, 0)^{(a,b)} = (a, b)^{-1}(l, 0)(a, b) = (-a - ab^{(-1)}, b^{(-1)})(l, 0)(a, b) = (lb + l, 0) \in E,$$

so E is a normal subgroup of H,

$$E \cap F = \{(e, 0)\}.$$

Hence $H = E \rtimes F$ is a semidirect product. Finally, the maps

$$\varphi : L \ni l \mapsto (l, 0) \in E \text{ and } \psi : T \ni t \mapsto (e, t) \in F$$

are group isomorphisms. □

Corollary 1. *A group $H(L, T)$ is abelian if and only if $LT = \{e\}$ and T is an abelian group.*

A non-empty set S is called a subbrace of of a brace A (see [6]) if the following hold:

s_1) $(S, +)$ is a subgroup of $(A, +)$,

s_2) $uv \in S$ for any $u, v \in S$.

It is obviously that $\{0\}$ and A are trivial subbraces in A. Since A can be regarded as A-module, every submodule I of A-module A is called a right ideal of A [5] (there is no similar concept of a left ideal). Therefore I is a right ideal of A if and only if the following hold:

i_1) $(I,+)$ is a subgroup of $(A,+)$,

i_2) $ia \in I$ for any $i \in I$ and $a \in A$.

If, moreover, I satisfies the condition

i_3) $ai \in I$ for any $i \in I$ and $a \in A$,

then I is called a two-sided ideal (for short an ideal) of A. Any (right or two-sided) ideal of A is a subbrace in A. For any brace A

- the left annihilator
$$\operatorname{ann}_l A = \{u \in A \mid uA = \{0\}\}$$
is a right ideal of A,

- the right annihilator
$$\operatorname{ann}_r A = \{v \in A \mid Av = \{0\}\}$$

is a two-sided ideal of A. In [5] $\operatorname{ann}_r A$ is denoted by $\operatorname{Soc}(A)$. Obviously that $\operatorname{ann} A = \operatorname{ann}_r A \cap \operatorname{ann}_l A$ is a two-sided ideal in A. Element $a \in A$ is called a left (respectively right) zero divisor if it satisfies the following two conditions:

z_1) $a \neq 0$,

z_2) $ab = 0$ (respectively $ca = 0$) for some non-zero element $b \in A$ (respectively $c \in A$).

Element $a \in A$ that is a left and a right zero divisor is called a zero divisor of A.

Remark 1. *If a brace A is left distributive, then A becomes a radical ring (i.e., an associative ring which is a group with respect to the circle operation "$\circ$"). The group $H(A^+, A^\circ)$, where A is a radical ring, was constructed by Ya.P.Sysak [1] and called the associated group of a radical ring A. Similarly, we will say that the group $H(A) = H(A^+, A^\circ)$ is associated with a brace A.*

Lemma 2. *Let A be a brace with the associated group $H(A) = E \rtimes F$. If S is a subbrace of A with the associated group $H(S) = U \rtimes W$, then the following conditions hold:*

(1) *$H(S) \leq H(A)$, $U \leq E$ and $W \leq F$,*

(2) *if S is a right ideal of A, then $U \lhd H(A)$,*

(3) *if S is an ideal of A, then $U \lhd H(A)$ and $H(S) \lhd H(A)$,*

(4) *if $U \lhd H(A)$, then $SA \subseteq S$,*

(5) *if $H(S) \lhd E \rtimes W$, then $AS \subseteq S$,*

(6) *the centralizers*

$$C_E(F) = \{(a,0) \in E \mid a \in \operatorname{ann}_l A\} \text{ and } C_F(E) = \{(0,u) \in F \mid u \in \operatorname{ann}_r A\};$$

in particular, is A not contains left and right zero divisors, then

$$C_F(E) = C_E(F) = \{(0,0)\}.$$

Proof. (1) follows from definition of $H(A)$.

(2) Let S be a right ideal of A. Then $sa \in S$ for any $s \in S$, $a \in A$ and so

$$(s,0)^{(a,b)} = (-a - ab^{(-1)}, b^{(-1)})(s,0)(a,b) = (sb+s,0) \in U \tag{2}$$

for any elements $(a,b) \in H(A)$ and $(s,0) \in U$. This means that U is a normal subgroup of $H(A)$.

(3) Assume that S is an ideal of A, $(a,b) \in H(A)$ and $(s,t) \in H(S)$. Then

$$\begin{gathered}(s,t)^{(a,b)} = (-(at)b - at - ((ab^{(-1)})t)b + sb + s - (ab^{(-1)})t,\\ t + tb + (b^{(-1)}t)b + b^{(-1)}t) \in H(S),\end{gathered}$$

and hence $H(S)$ is normal in $H(A)$.

(4) Since $U \lhd H(A)$, from (2) it follows that $sb+s \in S$ for any $s \in S$, $b \in A$ and therefore $sb \in S$.

(5) Let us $H(S) \lhd E \rtimes W$. Then for any $a \in A$ and $u,v,w \in S$ we deduce that

$$\begin{gathered}H(S) \ni (u,v)^{(a,w)} = \\ (-(av)w - ((aw^{(-1)})v)w - aw - (aw^{(-1)})w + uw - av - \\ (aw^{(-1)})v - aw^{(-1)} + u, w^{(-1)} \circ v \circ w) = \\ (-(av)w - ((aw^{(-1)})v)w + uw - av - (aw^{(-1)})v + u, v + vw + (w^{(-1)}v)w + w^{(-1)}v).\end{gathered}$$

If $w = 0$, then we obtain that $(-av+u,v) \in H(S)$, and so $AS \subseteq S$.

(6) Assume that $(a,0) \in C_E(F)$. Then $(a,0)(0,b) = (0,b)(a,0)$ for every $b \in A$ and consequently $ab = 0$. If $(0,u) \in C_F(E)$, then

$$(0 \cdot 0 + 0 + b, u \circ 0) = (0,u)(b,0) = (b,0)(0,u) = (bu + b + 0, 0 \circ u)$$

and therefore $bu = 0$. □

Lemma 3. *If A is a brace and $a,b \in A$, then $(a,b) \in Z(H(A))$ if and only if $aA = \{0\}$ and $Ab = \{0\} = bA$.*

Proof. ($\Rightarrow$) Let us $(a,b) \in Z(H(A))$. Then for any elements $u,v \in A$ we see that

$$(av + a + u, b \circ v) = (a,b)(u,v) = (u,v)(a,b) = (ub + u + a, v \circ b) \tag{3}$$

if and only if

$$\begin{aligned} b \circ v &= v \circ b, \\ av &= ub. \end{aligned}$$

Hence $bv = vb$. If $u = 0$, then $av = 0$ (and we obtain that $aA = \{0\}$). In the case $v = 0$ it follows that $ub = 0$ (and consequently $Ab = \{0\}$).

($\Leftarrow$) Since $ub = 0 = av$ for any $u,v \in A$, we conclude that $(u,v) \in Z(H(A))$. □

Remark 2. (1) *For any element a of a brace A the centralizers*

$$C_{A^\circ}(a) = \{z \in A^\circ \mid z \circ a = a \circ z\} \text{ and } C_A(a) = \{z \in A \mid za = az\}$$

are equal.

(2) *If*

$$Z(A) = \{z \in A \mid za = az \quad \text{for every} \quad a \in A\}$$

and

$$Z_1(A^\circ) = \{z \in A^\circ \mid z \circ a = a \circ z \quad \text{for every} \quad a \in A^\circ\},$$

then $Z(A) = Z_1(A^\circ)$ is a normal subgroup in A°.

Let S be a two-sided ideal of a brace A. On the set

$$A/S = \{a + S \mid a \in S\}$$

we have two operations "+" and "·" (see [6]) given by the rules:

- $(a_1 + S) + (a_2 + S) = (a_1 + a_2) + S$,
- $(a_1 + S) \cdot (a_2 + S) = (a_1 a_2) + S$ for $a_1, a_2 \in A$. Then $(A/S, +, \cdot)$ is a brace (and A/S is called the quotient brace of A with respect to an ideal S).

Lemma 4. *If S is an ideal of a brace A, then the groups $(A/S)^\circ$ and A°/S° are isomorphic.*

Proof. In fact, the rule

$$\varphi : (A/S)^\circ \ni a + S \mapsto a \circ S^\circ \in A^\circ/S^\circ$$

is a group isomorphism. □

Lemma 5. *If S is a two-sided ideal of a brace A, then the groups $H(A)/H(S)$ and $H(A/S)$ are isomorphic.*

Proof. Assume that $H(A) = E \rtimes F$, $H(S) = U \rtimes W$ and $H(A/S) = Q \rtimes R$. Then, by Lemma 2, we have that $U \leq E$, $W \leq F$ and so

$$H(A)/H(S) = (E \rtimes F)/H(S) \cong (EH(S)/H(S)) \rtimes (FH(S)/H(S)) =$$
$$= (EUW/UW) \rtimes (FUW/UW) \cong (EW/UW) \rtimes (FW/UW).$$

Furthermore, we have the group isomorphisms

$$Q \cong (A/S)^+ \cong A^+/S^+ \cong E/U \cong EW/UW$$

and, by Lemma 4,

$$R \cong (A/S)^\circ \cong A^\circ/S^\circ \cong F/W \cong FU/WU.$$

The lemma is prowed. □

2. Frobenius groups. Recall that a group $H = E \rtimes F$ is called a Frobenius group with a kernel E and a complement F if

$$F \cap F^g = \{1\}$$

for all $g \in H \setminus F$ and

$$E \setminus \{1\} = H \setminus \bigcup_{h \in H} F^h.$$

Proof of Theorem 1. Assume that $H = E \rtimes F$ is a Frobenius group with $E \cong L^+$ and $F \cong T$. By Lemma 1.1 of [3], for any elements $h \in T$ and $l \in L$ there exists $l_1 \in L$ such that $(l, 0) = [(l_1, 0), (e, h)]$ and consequently $(l, 0) = (l_1, 0)^{-1}(e, h)^{-1}(l_1, 0)(e, h) = (l_1 h, 0)$. Then $l = l_1 h$ and we conclude that $L = Lh$ for any $0 \neq h \in T$.

Suppose that $lt = e$ for some $t \in T$ and $0 \neq l \in L$. Then

$$\{(e, 0)\} = F \bigcap F^{(l,0)} \ni (e, t)^{(l,0)} = (-lt, 0), \tag{4}$$

which implies that $t = 0$ and $\operatorname{ann}_T l = \{0\}$.

($\Leftarrow$) Assume that a group $H = E \rtimes F$ satisfies the conditions (i) and (ii). If $0 \neq v \in T$ and $k \in L$, then $k = k_1 v$ for some $k_1 \in L$ and the commutator $[(k_1, 0), (0, e)] = (k_1 v, 0) = (k, 0)$ for any $(e, 0) \neq (e, t) \in F$. This means that $E = [E, (e, t)]$. Moreover, for any elements $(u, v) \in H$ and $(e, t) \in F$ we see that

$$F^{(u,v)} \ni (e, t)^{(u,v)} = (-(ut)v - ut - ((uv^{(-1)})t)v - (uv^{(-1)})t, v^{(-1)} \circ t \circ v).$$

If $v = 0$ and $u \neq 0$, then $(-ut, t) = (e, t)^{(u,0)} \in F^{(u,0)}$. This gives that

$$H \setminus \bigcup_{(u,v) \in H} F^{(u,v)} = E \setminus \{(e, 0)\}.$$

Now we assume that $(e, h)^{(u,v)} \in F \cap F^{(u,v)}$ for some $h, v \in T$ and $0 \neq u \in L$. Then

$$(e, h)^{(u,v)} = (-u(v^{(-1)} \circ h \circ v), v^{(-1)} \circ h \circ v),$$

and therefore $-u(v^{(-1)} \circ h \circ v) = e$. From this, in view of (ii), we have $v^{(-1)} \circ h \circ v = 0$ and consequently $h = 0$. Hence

$$F \bigcap F^{(u,v)} = \{(e, 0)\}$$

and H is a Frobenius group with a kernel E and a complement F. $\square$

Let R be an associative ring with 1, A be a right R-module. If $\mu : A \to U(R)$ is an additive map and

$$\mu(a\mu(b)) = \mu(a)$$

for all $a, b \in A$, then $(A, +, \cdot)$ is a brace (see Example 1 of [5]) with a multiplication "$\cdot$" given by the rule

$$ab = a(\mu(b) - 1) \tag{5}$$

Example 1. *As in Example 3 of [5], $A = \{0,1,2,3,4,5\}$, $A^+ \cong \mathbb{Z}_6$ and $\mu : A \to U(\mathbb{Z}_6)$ is such that*

$$\mu(0) = \mu(2) = \mu(4) = 1,$$
$$\mu(1) = \mu(3) = \mu(5) = 0.$$

Then $(A, +, \cdot)$ is a brace with the multiplication given by (5) (and depicted by Table):

$\cdot$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	4	0	4	0	4
2	0	2	0	2	0	2
3	0	0	0	0	0	0
4	0	4	0	4	0	4
5	0	2	0	2	0	2

If $L = \{0,2,4\}$, then $LA = L$ and so L is an A-module. Since $T = \{0,5\}$ is a subgroup in A°,

$$L \cdot 5 = \{0 \cdot 5, 2 \cdot 5, 4 \cdot 5\} = \{0,2,4\} = L$$

and

$$\operatorname{ann}_T 2 = \{0\} = \operatorname{ann}_T 4,$$

we conclude that $H(L,T)$ is a Frobenius group.

3. Nilpotent braces. In this section we investigate the properties of nilpotent braces.

Lemma 6. *If A is a brace and $k > 0$, then $A^{(k+1)}$ is an ideal of $A^{(k)}$.*

Proof. It is easy to seen that $A^{(k+1)}$ is a subgroup of $A^{(k)}$. Since

$$A^{(k+1)}A^{(k)} \subseteq A^{(k+1)}A \subseteq A^{(k+2)} \subseteq A^{(k+1)}$$

and

$$A^{(k)}A^{(k+1)} \subseteq A^{(k)}A \subseteq A^{(k+1)},$$

we obtain the result. □

Remark 3. *Let A be a brace and p a prime. Then*

(1) *$H(A)$ is a torsion group if and only if A^+ and A° are torsion;*

(2) *$H(A)$ is a p-group if and only if A^+ and A° are p-groups.*

As in Lemma 2.4 of [2] we can prove the next

Theorem 3. *Let A be a right nilpotent (respectively left nilpotent) brace, p a prime. Then*

(1) *A^+ is a p-group if and only if A° is a p-group;*

(2) *A^+ is a torsion-free group if and only if A° is a torsion-free group.*

Proof. (a) Asuume that A is a right nilpotent brace of index n. We prove by induction on n. Since
$$A^{(n-1)}A^{(n-1)} \subseteq A^{(n-1)}A = \{0\},$$
we conclude that $A^{(n-1)}$ is a commutative radical ring. Now we assume that the result is true for right nilpotent braces of index $< n$. Since
$$(A/A^{(2)})^{(2)} \subseteq (A/A^{(2)}) \cdot (A/A^{(2)}) = A^{(2)}/A^{(2)} = \{\bar{0}\},$$
we have group isomorphisms
$$(A/A^{(2)})^+ \cong (A/A^{(2)})^\circ \cong A^\circ/(A^{(2)})^\circ$$
and the assertion follows.

(b) For arbitrary k, A^k is an ideal of A,
$$(A^k/A^{k+1})^+ \cong (A^k/A^{k+1})^\circ$$
and for a left nilpotent brace A the assertion is also true. □

Lemma 7. *Let A be a brace. Then $Z(H(A)) \neq \{(0,0)\}$ if and only if $\operatorname{ann}_l A \neq \{0\}$.*

Proof. $(\Leftarrow)$ If $0 \neq a \in \operatorname{ann}_l A$, then, by Lemma 2, $(a,0) \in C_E\ (F)$ and therefore
$$(0,0) \neq (a,0) \in Z(H(A)).$$

$(\Rightarrow)$ If $(a,0) \in Z(H(A))$ for some $0 \neq a \in A$, then for any elements $u, v \in A$ we obtain
$$(av + a + u, v) = (a,0)(u,v) = (u,v)(a,0) = (u+a, v).$$
This yields that $av = 0$ and so $a \in \operatorname{ann}_l A$. □

Corollary 2. *Let A be a brace. If $H(A) = E \rtimes F$ and $Z(H(A)) \not\subseteq E$, then*
$$Z(A) \cap \operatorname{ann}_r A \neq \{0\}.$$

Proof. Assume that $(a,b) \in Z(H(A))$ and $b \neq 0$. Then for any $u \in A$ we have
$$(a+u, b) = (a,b)(u,0) = (u,0)(a,b) = (ub + u + a, b)$$
and
$$(au + a, b \circ u) = (a,b)(0,u) = (0,u)(a,b) = (a, u \circ b)$$
and consequently $ub = 0$, $au = 0$ and $b \circ u = u \circ b$. This yields that $b \in Z(A)$. □

4. Proof of Theorem 2. (1) Let A be a non-zero left nilpotent brace of index n. Then $(A^{n-1})A = \{0\}$ and $A^{n-1} \neq \{0\}$. This means that $A^{n-1} \subseteq \operatorname{ann}_l A$ and, by Lemma 7, $Z(H(A)) \neq \{(0,0)\}$. Since A^{n-1} is a two-sided ideal in A and

$$(A/A^{n-1})^{n-1} = \{\bar{0}\},$$

by induction on n we can prove that $H(A)$ is a nilpotent group. Moreover, $Z(A^\circ) \lhd A^\circ$ and so

$$\{0\} \neq (A^{n-1})^\circ \bigcap Z(A^\circ) \subseteq \operatorname{ann} A.$$

(2) We have $A^{(n)} = \{0\}$ for some positive integer n and thus

$$A^{(n-1)} \subseteq \operatorname{ann}_r A.$$

But $\operatorname{ann}_r A$ is a two-sided ideal in A and so $(\operatorname{ann}_r A)^\circ$ is an abelian normal subgroup of A°. By induction on n we obtain the result. □

Example 2. *Let $(\mathbb{F}_2)^3$ be a brace constructed in* [5] *(see Example 2) with the multiplication "$\cdot$" depicted by Table:*

$\cdot$	000	111	100	011	010	101	001	111
000	000	000	000	000	000	000	000	000
111	000	000	000	000	000	000	000	000
100	000	110	000	001	111	110	111	001
011	000	110	000	001	111	110	111	001
010	000	110	111	110	000	001	111	001
101	000	110	111	110	000	001	111	001
001	000	000	111	111	111	111	000	000
111	000	000	111	111	111	111	000	000

This brace has a series

$$A \supset A^{(2)} = \{000, 111, 001, 100\} \supset A^{(3)} = \{000, 111\} \supset A^{(4)} = \{000\}$$

and

$$A \supset A^2 = \{000, 111, 001, 100\} = A^3.$$

This means that A is right nilpotent and A is not left nilpotent. Since

$$111 \cdot 100 = 000 \neq 110 = 100 \cdot 111,$$

we conclude that $111 \notin Z(A)$ and so $A^{(3)} \cap Z(A) = \{000\}$. If $a = 001$, $b = 100$, then

$$A^\circ = \langle a \rangle \rtimes \langle b \rangle$$

is a dihedral group of order 8. Hence $H(A)$ is a 2-group of order 64 and it is nilpotent. If $H(A) = E \rtimes F$, then, by Corollary 2, we have $Z(H(A)) \subseteq E$.

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У данiй статтi побудовано групу $H(A)$, асоцiйовану з брейсом A, i дослiджено її властивостi.

Артемович О.Д., Скаскив Л.В. *Группы ассоциированные с брейсами* // Карпатские математические публикации. — 2011. — Т.3, №1. — С. 4–14.

В этой роботе построено группу $H(A)$, ассоциированную с брейсом A, и исследовано её свойства.