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ON OPERATIONS ON SOME CLASSES OF DISCONTINUOUS MAPS

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A map $f : X \to Y$ between topological spaces is called scatteredly continuous (pointwise discontinuous) if for each non-empty (closed) subspace $A \subset X$ the restriction $f|_A$ has a point of continuity. We define a map $f : X \to Y$ to be weakly discontinuous if for every non-empty subspace $A \subset X$ the set $D(f|_A)$ of discontinuity points of the restriction $f|_A$ is nowhere dense in A.

In this paper we consider the composition, Cartesian and diagonal product of weakly discontinuous, scatteredly continuous and pointwise discontinuous maps.

Introduction

A map $f : X \to Y$ between topological spaces is called scatteredly continuous if for each non-empty subspace $A \subset X$ the restriction $f|_A$ has a point of continuity. Such maps were introduced in [1] and were more investigated in [3].

By its spirit definition of a scatteredly continuous map resembles the classical definition of a pointwise discontinuous map, due to R.Baire [2]. We recall that the map $f : X \to Y$ is called pointwise discontinuous if for each non-empty closed subspace $A \subset X$ the restriction $f|_A$ has a continuity point.

Following [6] we define a map $f : X \to Y$ to be weakly discontinuous if for every subspace $A \subset X$ the set $D(f|_A)$ of discontinuity points of the restriction $f|_A$ is nowhere dense in A.

In this paper we consider the composition, Cartesian and diagonal product of weakly discontinuous, scatteredly continuous and pointwise discontinuous maps. In particular, one of the main results of the paper is the following theorem.

Theorem 1. *Let $\mathcal{F} = \{f_\alpha\}_{\alpha\in S}$ be a family of maps f_α of a topological space X_α into a topological space Y_α respectively. The Cartesian product $\prod\limits_{\alpha\in S} f_\alpha : \prod\limits_{\alpha\in S} X_\alpha \to \prod\limits_{\alpha\in S} Y_\alpha$ is a scatteredly continuous map if and only if the following conditions hold:*

(i) all the maps f_α are scatteredly continuous;

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(ii) all the maps f_α, except maybe one, are weakly discontinuous;
(iii)all the maps f_α, except maybe finite number, are continuous.

Also we show that the Cartesian and diagonal product of finite number of weakly discontinuous maps is weakly discontinuous.

1 Preliminaries

A "space" always means "topological space". By $\mathbb{R}$ and $\mathbb{Q}$ we denote the spaces of real and rational numbers respectively; ω stands for the space of finite ordinal numbers (=non-negative integer numbers) endowed with the discrete topology.

For a subset A of a topological space X by $cl_X(A)$ or $\overline{A}$ we denote the closure of A in X while $Int_X(A)$ stands for the interior of A in X.

For a map $f : X \to Y$ between topological spaces by $C(f|_A)$ we denote the set of continuity points of the restriction $f|_A$ while $D(f|_A)$ stands for the discontinuity points of the restriction $f|_A$.

The characteristic function of a subset A of a set X is a function $\chi_A : X \to \{0, 1\}$ defined as follows

$$\chi_A(x) = \begin{cases} 1, & x \in A; \\ 0, & x \notin A. \end{cases}$$

Suppose we are given a family $\{X_s : s \in S\}$ of topological spaces. We consider a Cartesian product $X = \prod\limits_{s\in S} X_s$ of the sets $\{X_s : s \in S\}$ with Tychonoff topology. By $\pi_{X_s} : \prod\limits_{s\in S} X_s \to X_s$ we denote the projection of $X = \prod\limits_{s\in S} X_s$ onto X_s.

All spaces encountered in this paper (unless stated otherwise) are assumed to be Hausdorff. The rest of the notation and terminology is standard and can be found in [4].

2 Some facts about scatteredly continuous, weakly discontinuous and pointwise discontinuous maps

Definition 2.1. *A map $f : X \to Y$ between topological spaces is called*

- *weakly discontinuous if for each non-empty subspace $A \subset X$ the set $D(f|_A)$ is nowhere dense in A;*
- *scatteredly continuous if for each non-empty subspace $A \subset X$ the restriction $f|_A$ has a point of continuity;*
- *pointwise discontinuous (see [2]) if for each non-empty closed subspace $A \subset X$ the restriction $f|_A$ has a point of continuity.*

Obviously, every weakly discontinuous map is scatteredly continuous and each scatteredly continuous map is pointwise discontinuous.

As an example of scatteredly continuous, not a weakly discontinuous map one can take an identity map $f : \mathbb{R} \to \mathbb{R}_{\mathbb{Q}}$ from the real line equipped with the standard topology τ to the real line endowed with the topology generated by the subbase $\tau \cup \{\mathbb{Q}\}$. In [1] it is proved, in particular, that scatteredly continuous map $f : X \to Y$ into a regular space Y is weakly discontinuous.

Recall that the Riemann function is a function $R : [0,1] \to [0,1]$ defined as follows

$$R(x) = \begin{cases} \frac{1}{n}, \text{ if } x = \frac{m}{n} \text{ is a rational number;} \\ 0, \text{ if } x \text{ is irrational.} \end{cases}$$

Obviously, the Riemann function is an example of pointwise discontinuous, not a scatteredly continuous map.

Lemma 2.1. *Let $f : X \to Y$ be a scatteredly continuous map. Then for each non-empty subspace $A \subset X$ the set $C(f|_A)$ is dense in A.*

Proof. Without loss of generality we can assume that $A = X$. Put $X_0 = \{x \in X : f : X \to Y$ is continuous at the point $x\}$. We prove that $\overline{X_0} = X$. Suppose this is not true, that is $X \backslash \overline{X_0} \neq \varnothing$. Put $U = X \backslash \overline{X_0}$ and let x_0 be a continuity point of the restriction $f|_U : U \to Y$. Then for any neighborhood $O(f(x_0))$ of the point $f(x_0)$ there is a neighborhood $O(x_0)$ of the point x_0 such that $f(O(x_0) \cap U) \subset O(f(x_0))$. Since the set U is open in X so it is the set $O(x_0) \cap U$. Therefore, $f : X \to Y$ is continuous at x_0, hence $x_0 \in X_0$, which contradicts the fact that $x_0 \in X \backslash \overline{X_0}$. □

Proposition 2.1. *A map $f : X \to Y$ is scatteredly continuous if and only if there is an ordinal number β_0 and a pairwise disjoint family $\{X_\alpha\}_{\alpha<\beta_0}$ of non-empty subsets of X such that $X = \cup\{X_\alpha : \alpha < \beta_0\}$; for each $\beta < \beta_0$ the set X_β is dense in the subspace $\cup\{X_\alpha : \beta \leq \alpha < \beta_0\}$ and $C(f|_{\cup\{X_\alpha : \beta\leq\alpha<\beta_0\}}) = X_\beta$. The ordinal number β_0 is called an index of scattered continuity of the map f and is denoted by $sc(f)$.*

Proof. The "only if" part. Let $f : X \to Y$ be a scatteredly continuous map. Apply a transfinite induction to all ordinal numbers which are less than $|X|^+$. Put $X_0 = \{x \in X : f : X \to Y$ is continuous at a point $x\}$. Then, by Lemma 2.1, $\overline{X_0} = X$. Put $X^0 = X \backslash X_0$ and $X_1 = \{x \in X^0 : f|_{X^0} : X^0 \to Y$ is continuous at a point $x\}$. Due to Lemma 2.1 $\overline{X_1} = X \setminus X_0$. Put $X^1 = X^0 \setminus X_1$ and so on. Suppose that for each ordinal number $\alpha < \beta$ we have constructed the sets X_α and X^α. Then put $P^\beta = \bigcap\{X^\alpha : \alpha < \beta\}$, $X_\beta = \{x \in P^\beta : f|_{P^\beta} : P^\beta \to Y$ is continuous at a point $x\}$ and $X^\beta = P^\beta \backslash X_\beta$. By β_0 we denote a minimal ordinal number β such that $X_\beta = \varnothing$. Since f is a scatteredly continuous map, $X = \bigcup\{X_\alpha : \alpha < \beta_0\}$. Obviously, for any ordinal number $\beta < \beta_0$ the restriction $f|_{\cup\{X_\alpha:\beta\leq\alpha<\beta_0\}} : \cup\{X_\alpha : \beta \leq \alpha < \beta_0\} \to Y$ is continuous at each point of the set X_β, and for all $\beta < \beta_0$ the set X_β is dense in $\bigcup\{X_\alpha : \beta \leq \alpha < \beta_0\}$.

The "if" part. Suppose there is an ordinal number β_0 and a pairwise disjoint family $\{X_\alpha\}_{\alpha<\beta_0}$ of non-empty subsets of X such that $X = \cup\{X_\alpha : \alpha < \beta_0\}$ and for each $\beta < \beta_0$ $C(f|_{\cup\{X_\alpha:\beta\leq\alpha<\beta_0\}}) = X_\beta$. Let A be a non-empty subset of X. Put $\alpha_0 = min\{\alpha : A \cap X_\alpha \neq \varnothing\}$. Then $C(f|_A) \supset A \cap X_{\alpha_0} \neq \varnothing$. □

Proposition 2.2. *A map $f : X \to Y$ is weakly discontinuous if and only if there is an ordinal number β_0 and a pairwise disjoint family $\{X_\alpha\}_{\alpha<\beta_0}$ of non-empty subsets of X such that $X = \cup\{X_\alpha : \alpha < \beta_0\}$; for each $\beta < \beta_0$ the set X_β is an open dense subset of*

$\cup\{X_\alpha : \beta \le \alpha < \beta_0\}$ *and* $Int_{\cup\{X_\alpha:\beta\le\alpha<\beta_0\}}C(f|_{\cup\{X_\alpha:\beta\le\alpha<\beta_0\}}) = X_\beta$. *The ordinal number* β_0 *is called an index of weak discontinuity of the map* f *and is denoted by* $wd(f)$.

Proof. Is similar to the proof of Proposition 2.1. □

Proposition 2.3. *The composition of two weakly discontinuous maps is weakly discontinuous.*

Proof. Let $f : X \to Y, g : Y \to Z$ be two weakly discontinuous maps. To show that $g \circ f$ is weakly discontinuous, it suffices, given a non-empty subspace $A \subset X$ to find a non-empty open subset $U \subset A$ such that $g \circ f|_U$ is continuous. The weak discontinuity of f yields a non-empty open set $V \subset A$ such that $f|_V$ is continuous. The weak discontinuity of g yields a non-empty open subset $W \subset f(V)$ such that $g|_W$ is continuous. By the continuity of $f|_V$, the preimage $U = (f|_V)^{-1}(W)$ is open in V and hence in A. Finally, the continuity of the functions $f|_U$ and $g|_{f(U)}$ imply the continuity of $g \circ f|_U$. □

Proposition 2.4. *The composition* $g\circ f : X \to Z$ *of a weakly discontinuous map* $f : X \to Y$ *and a scatteredly continuous map* $g : Y \to Z$ *is scatteredly continuous.*

Proof. Given a non-empty subspace $A \subset X$ we should find a continuity point of $g \circ f|_A$. The weak discontinuity of f implies the existence of a non-empty open set $V \subset A$ such that $f|_V$ is continuous. The scattered continuity of g implies the existence of continuity point $y_0 \in f(V)$ of the restriction $g|_{f(V)}$. Then any point $x_0 \in (f|_V)^{-1}(y_0)$ is a continuity point of $g \circ f|_A$. □

However, the composition $g \circ f : X \to Z$ of weakly discontinuous (even more that, continuous) map $f : X \to Y$ and a pointwise discontinuous map $g : Y \to Z$ need not be pointwise discontinuous.

Example 1. *We consider the identity maps* $i_1 : (X, \tau_0) \to (X, \tau_z)$, $i_2 : (X, \tau_z) \to (X, \tau_s)$ *where* $X = [0; 1)$, τ_z *is a standard topology,* τ_s *is a right half-open interval topology and* τ_0 *is a topology generated by the subbase* $\tau_z \cup \{0\}$. *Obviously, the map* $i_1 : (X, \tau_0) \to (X, \tau_z)$ *is continuous.*

The map i_2 *is pointwise discontinuous. Assume that* A *is a non-empty closed subset of* X. *As the point of continuity of the restriction* $i_2|_A$ *we can take the minimal point of the set* A *with respect to standard order on the set* $[0, 1)$.

Since the restriction $i_2|_{(0,1)}$ *is everywhere discontinuous, the map* i_2 *fails to be scatteredly continuous.*

The set $A = (0, 1)$ *is closed in* (X, τ_0). *However, the restriction* $(i_2 \circ i_1)|_A$ *has no point of continuity. Thus the composition* $i_2 \circ i_1$ *is not a pointwise discontinuous map.*

Proposition 2.5. *The composition* $g \circ f : X \to Z$ *of closed continuous map* $f : X \to Y$ *and a pointwise discontinuous map* $g : Y \to Z$ *is pointwise discontinuous.*

It is interesting to note that the composition $g \circ f : X \to Z$ of a scatteredly continuous map $f : X \to Y$ and a weakly discontinuous map $g : Y \to Z$ can be everywhere discontinuous.

Example 2. *Let* $f : \mathbb{R} \to \mathbb{R}_{\mathbb{Q}}$ *be the identity map from the real line equipped with the standard topology* τ *to the real line endowed with the topology generated by the subbase* $\tau \cup \{\mathbb{Q}\}$*. Also let* $\chi_{\mathbb{Q}} : \mathbb{R}_{\mathbb{Q}} \to \{0; 1\}$ *be the characteristic function of the set* $\mathbb{Q}$*. It is easy to show that the map* $f : \mathbb{R} \to \mathbb{R}_{\mathbb{Q}}$ *is scatteredly continuous and* $\chi_{\mathbb{Q}} : \mathbb{R}_{\mathbb{Q}} \to \{0; 1\}$ *is a weakly discontinuous map while their composition* $\chi_{\mathbb{Q}} \circ f : \mathbb{R} \to \{0; 1\}$ *is everywhere discontinuous.*

Proposition 2.6. *The composition* $g \circ f : X \to Z$ *of a pointwise discontinuous (scatteredly continuous, weakly discontinuous) map* $f : X \to Y$ *and a continuous map* $g : Y \to Z$ *is pointwise discontinuous (scatteredly continuous, weakly discontinuous respectively).*

Proposition 2.7. *Let* f *be a surjective open map from a topological space* X *onto a topological space* Y *and* g *be a map from the space* Y *to some topological space* Z*. Then scattered continuity (weak discontinuity) of the map* $g \circ f$ *implies scattered continuity (weak discontinuity) of the map* g*.*

Proof. Let f be an open surjective map and let the composition $g \circ f$ be a scatteredly continuous (weakly discontinuous respectively) map. Assume that B is a non-empty subset of Y and $A = f^{-1}(B)$. It is known that the restriction $f|_A : A \to B$ is an open map. Since the composition $g \circ f$ is scatteredly continuous (weakly discontinuous), the set $C(g \circ f|_A) \neq \varnothing$ ($C((g \circ f)|_A)$ is an open subset of A respectively). We take some $x_0 \in C(g \circ f|_A)$ and show that the map $g|_B : B \to Z$ is continuous at the point $y_0 = f(x_0)$. Assume that $O(g(y_0))$ is a neighborhood of the point $g(y_0)$ in Z. Since $g \circ f(x_0) = g(y_0)$ and $x_0 \in C(g \circ f|_A)$, there is a neighborhood $O(x_0)$ of the point x_0 such that $g \circ f(O(x_0) \cap A) \subset O(g(y_0))$. Since the restriction $f|_A : A \to B$ is an open map, the set $f(O(x_0) \cap A)$ is an open subset of B with $y_0 \in f(O(x_0) \cap A)$. It is easy to understand that $g(f(O(x_0) \cap A)) = g \circ f(O(x_0) \cap A) \subset O(g(y_0))$.

If the composition $g \circ f$ is weakly discontinuous, the set $f(C((g \circ f)|_A))$ is an open subset of B. □

If the map f is surjective open and the composition $g \circ f$ is a pointwise discontinuous map, then the map g need not be pointwise discontinuous.

Example 3. *We consider the identity maps* $f : (X, \tau_z) \to (X, \tau_0)$*,* $g : (X, \tau_0) \to (X, \tau_s)$ *where* $X = [0; 1)$*,* τ_z *is a standard topology,* τ_s *is a right half-open interval topology and* τ_0 *is a topology generated by the subbase* $\tau_z \cup \{0\}$*. Obviously, the map* f *is open. As Example 1 shows, the composition* $g \circ f : (X, \tau_z) \to (X, \tau_s)$ *is a pointwise discontinuous map, but the map* $g : (X, \tau_0) \to (X, \tau_s)$ *is not a pointwise discontinuous map.*

Proposition 2.8. *Let* f *be a surjective open continuous map from a topological space* X *onto a topological space* Y *and* g *be a map from the space* Y *to some topological space* Z*. Then pointwise discontinuity of the map* $g \circ f$ *implies a pointwise discontinuity of the map* g*.*

Propositions 2.7 and 2.8 are not faithful, if we replace the openess of the map f by the quotientity (even more that, by closedness and continuity).

Example 4. *Let X be a scattered space, Y be a perfect non-scattered space and $f : X \to Y$ be a closed surjective continuous map (such spaces exist, see[5]). By Y_d we denote the set Y endowed with the discrete topology. Let g be the identity map from Y to Y_d. Obviously, the composition $g \circ f : X \to Y_d$ is scatteredly continuous (even weakly discontinuous), but the map g is everywhere discontinuous.*

Proposition 2.9. *Let f be a surjective perfect map from a topological space X onto a topological space Y and g be a map from the space Y to some topological space Z. Then weak discontinuity of the map $g \circ f$ implies weak discontinuity of the map g.*

Proof. Let f be an surjective perfect map and let the composition $g \circ f$ be a weakly discontinuous map. Let B be some non-empty subset of Y. Put $P = \overline{B}$. Since the map f is perfect, there is a closed subset F of X such that $f(F) = \overline{B}$ and $f|_F$ is irreducible perfect map. Since the composition $g \circ f$ is weakly discontinuous map, there is an open subset U of F of the points of continuity of the restriction $g \circ f|_F$. Since $f|_F$ is irreducible map, $f(U)$ is non-empty open subset of P. And since $\overline{B} = P$, $f(U) \cap B$ is a non-empty open set of the continuity points of the restriction $g|_B$. □

Recall that a space X is called a Preiss-Simon space if for an arbitrary non-empty closed subset A of X and each point $x \in A$ there is a sequence $\{U_n : n \in \omega\}$ of non-empty open subsets of A that converges to x in the sense that each neighborhood of x contains all but finitely many sets U_n.

Proposition 2.10. *Let f be a closed surjective map from a perfectly paracompact space X onto a hereditary Baire Preiss-Simon space Y and g be a map from the space Y to a regular space Z. Then scattered continuity of the map $g \circ f$ implies scattered continuity of the map g.*

Proof. In [3], in particular, it is proved that a map g from a hereditary Baire Preiss-Simon space Y to a regular space Z is scatteredly continuous if for any open subset in Z its preimage is a G_δ-set in Y. Suppose g is not a scatteredly continuous map. Then there is an open set U in Z such that $g^{-1}(U)$ is not G_δ-set in Y.

From the other hand, as $g \circ f$ is a scatteredly continuous map from a perfectly paracompact space X to a regular space Z, then $(g \circ f)^{-1}(U)$ is a G_δ-set in X (see [3]).

Put $A = (g \circ f)^{-1}(U) \subset X$. Then $f(A) = g^{-1}(U)$. Since A is a G_δ-set in X, then $X \backslash A$ is an F_σ-set in X, that is, $X \backslash A = \bigcup\{F_i : i \in \omega\}$ with F_i – close subset in X for all $i \in \omega$. Then $f(X \backslash A) = \bigcup f(F_i)$ is an F_σ-set in Y. But then $Y \backslash f(X \backslash A) = g^{-1}(U)$ is a G_δ-set in Y, which is a contradiction. □

The next example shows that the closedness of the map f in Lemma 2.10 is essential.

Example 5. *Assume that f is a map from a scattered continuum compact X to the segment $Y = [0, 1]$, and $\chi_{\mathbb{Q}} : [0, 1] \to \mathbb{R}$ is the characteristic function of the set $\mathbb{Q}$. Spaces X and Y are both compact. Obviously, maps $\chi_{\mathbb{Q}} \circ f : X \to \mathbb{R}$ and f are scatteredly continuous. But the characteristic function $\chi_{\mathbb{Q}} : [0, 1] \to \mathbb{R}$ is everywhere discontinuous.*

3 Cartesian and diagonal product of maps

Suppose we are given two families $\{X_\alpha\}_{\alpha\in S}$ and $\{Y_\alpha\}_{\alpha\in S}$ of topological spaces and a family of maps $\{f_\alpha\}_{\alpha\in S}$, where $f_\alpha : X_\alpha \to Y_\alpha$. The map assigning to the point $x = \{x_\alpha\}_{\alpha\in S} \in \prod\limits_{\alpha\in S} X_\alpha$ the point $\{f_\alpha(x_\alpha)\}_{\alpha\in S} \in \prod\limits_{\alpha\in S} Y_\alpha$ is called the *Cartesian product of the maps* $\{f_\alpha\}_{\alpha\in S}$ and is denoted by $\prod\limits_{\alpha\in S} f_\alpha$ or $f_1 \times f_2 \times \cdots \times f_k$ if $S = \{1, 2, ..., k\}$.

Suppose we are given a topological space X, a family $\{Y_\alpha\}_{\alpha\in S}$ of topological spaces and a family of maps $\{f_\alpha\}_{\alpha\in S}$, where $f_\alpha : X \to Y_\alpha$. The map assigning to the point $x \in X$ the point $\{f_\alpha(x)\}_{\alpha\in S} \in \prod\limits_{\alpha\in S} Y_\alpha$ is called the *diagonal product of the maps* $\{f_\alpha\}_{\alpha\in S}$ and is denoted by $\mathop{\triangle}\limits_{\alpha\in S} f_\alpha$, or by $f_1 \triangle f_2 \triangle \ldots \triangle f_k$ if $S = \{1, 2, ..., k\}$.

Proposition 3.1. *Suppose we are given a topological space X, a family $\{Y_\alpha\}_{\alpha\in S}$ of topological spaces and a family of maps $\{f_\alpha\}_{\alpha\in S}$, where $f_\alpha : X \to Y_\alpha$. If the Cartesian product $\prod\limits_{\alpha\in S} f_\alpha : X^S \to \prod\limits_{\alpha\in S} Y_\alpha$ is a pointwise discontinuous (scatteredly continuous, weakly discontinuous) map, then so is the diagonal product $\mathop{\triangle}\limits_{\alpha\in S} f_\alpha : X \to \prod\limits_{\alpha\in S} Y_\alpha$.*

Proof. We consider the homeomorphic embedding $i : X \to X^S$. Obviously, $(\prod\limits_{\alpha\in S} f_\alpha) \circ i = \mathop{\triangle}\limits_{\alpha\in S} f_\alpha$. Propositions 2.5, 2.4 and 2.3 complete the proof. □

Proposition 3.2. *Suppose we are given two families $\{X_\alpha\}_{\alpha\in S}$ and $\{Y_\alpha\}_{\alpha\in S}$ of topological spaces and a family of maps $\{f_\alpha\}_{\alpha\in S}$, where $f_\alpha : X_\alpha \to Y_\alpha$. If the map $\prod\limits_{\alpha\in S} f_\alpha : \prod\limits_{\alpha\in S} X_\alpha \to \prod\limits_{\alpha\in S} Y_\alpha$ is pointwise discontinuous (scatteredly continuous, weakly discontinuous), then for each $\alpha \in S$ the map $f_\alpha : X_\alpha \to Y_\alpha$ is pointwise discontinuous (scatteredly continuous, weakly discontinuous).*

Proof. Let $\prod\limits_{\alpha\in S} f_\alpha$ be a pointwise discontinuous (scatteredly continuous, weakly discontinuous) map. For all $\alpha \in S$ the equality $\pi_{Y_\alpha} \circ (\prod\limits_{\alpha\in S} f_\alpha) = f_\alpha \circ \pi_{X_\alpha}$ holds. By Proposition 2.6, the composition $\pi_{Y_\alpha} \circ (\prod\limits_{\alpha\in S} f_\alpha)$ is pointwise discontinuous (scatteredly continuous, weakly discontinuous) for each $\alpha \in S$. Then the composition $f_\alpha \circ \pi_{X_\alpha}$ is pointwise discontinuous (scatteredly continuous, weakly discontinuous) for each $\alpha \in S$ as well. Since the map $\pi_{X_\alpha} : \prod\limits_{\alpha\in S} X_\alpha \to X_\alpha$ is open continuous and surjective for all $\alpha \in S$, due to Proposition 2.8 (Proposition 2.7 respectively), the map $f_\alpha : X_\alpha \to Y_\alpha$ is pointwise discontinuous (scatteredly continuous, weakly discontinuous) for each $\alpha \in S$. □

We show that the Cartesian product of two pointwise discontinuous maps need not be a pointwise discontinuous map.

Example 6. *We consider the Cartesian product $i_2 \times i_2 : (X, \tau_z) \times (X, \tau_z) \to (X, \tau_s) \times (X, \tau_s)$ of the pointwise discontinuous map $i_2 : (X, \tau_z) \to (X, \tau_s)$ (see Example 1) with itself. Put $F = \{(x; -x + 1) : x \in X\}$. The set F is closed in the space $(X, \tau_z) \times (X, \tau_z)$ and is a discrete subspace of the space $(X, \tau_s) \times (X, \tau_s)$. The restriction $(i_2 \times i_2)|_F$ has no points of continuity, otherwise this point would be isolated in F.*

Lemma 3.1. *Suppose we are given two maps $f : X \to S$ and $g : Y \to T$, where X, Y, S and T are topological spaces. Then the map $f \times g : X \times Y \to S \times T$ is scatteredly continuous if and only if the maps f and g are both scatteredly continuous and at least one of them is weakly discontinuous.*

Proof. The "if" part. Let $f : X \to S$ be a weakly discontinuous map and let $g : Y \to T$ be a scatteredly continuous map. Since g is a scatteredly continuous map, by Proposition 2.1, there is an ordinal number $sc(g)$ and a pairwise disjoint family $\{Y_\gamma\}_{\gamma < sc(g)}$ of non-empty subsets of Y such that the space $Y = \cup\{Y_\gamma : \gamma < sc(g)\}$; for each $\gamma < sc(g)$ the set Y_γ is dense in $\cup\{Y_\lambda : \gamma \leq \lambda < sc(g)\}$ and $C(g|_{\cup\{Y_\lambda : \gamma \leq \lambda < sc(g)\}}) = Y_\gamma$. Since f is a weakly discontinuous map, due to Proposition 2.2, there is an ordinal number $wd(f)$ and a pairwise disjoint family $\{X_\alpha\}_{\alpha < wd(f)}$ of non-empty subsets of X such that $X = \cup\{X_\alpha : \alpha < wd(f)\}$; for each $\alpha < wd(f)$ the set X_α is open dense subset of $\cup\{X_\beta : \alpha \leq \beta < wd(f)\}$ and $Int_{\cup\{X_\beta : \alpha \leq \beta < wd(f)\}} C(f|_{\cup\{X_\beta : \alpha \leq \beta < wd(f)\}}) = X_\alpha$. Take any set $A \subset X \times Y$. Put $A_1 = \pi_X(A)$ and $A_2 = \pi_Y(A)$. Without loss of generality, we can assume that $A_1 \cap X_0 \neq \varnothing$. For each $x \in X_0 \cap A_1$ by $\gamma(x)$ we denote a minimal ordinal number γ such that $\pi_Y(\pi_X^{-1}(x) \cap A) \cap Y_\gamma \neq \varnothing$. And put $\gamma_0 = min\{\gamma(x) : x \in X_0 \cap A_1\}$. Since $X_0 \cap A_1 \neq \varnothing$ and $Y_{\gamma_0} \cap A_2 \neq \varnothing$, $X_0 \times Y_{\gamma_0} \cap A \neq \varnothing$. We show that each point of the set $X_0 \times Y_{\gamma_0} \cap A$ is a continuity point of the restriction $f \times g|_A$. Take an arbitrary point $(x_0; y_0) \in X_0 \times Y_{\gamma_0} \cap A$. We fix some neighborhood $O(f \times g(x_0; y_0)) = O(f(x_0), g(y_0)) = O(f(x_0)) \times O(g(y_0))$ of the point $(f(x_0), g(y_0))$. We needed to show that there is a neighborhood $O(x_0, y_0) = O(x_0) \times O(y_0)$ of the point (x_0, y_0) such that $f \times g(O(x_0) \times O(y_0) \cap A) \subset O(f(x_0)) \times O(g(y_0))$. Since X_0 is an open subset of X and the map f is continuous at the point $x_0 \in X_0$ there is a neighborhood $O(x_0)$ of the point x_0 such that $O(x_0) \subset X_0$ and $f(O(x_0)) \subset O(f(x_0))$. Suppose that for all neighborhoods $O(y_0)$ of the point y_0 in Y we have that $f \times g(O(x_0) \times O(y_0) \cap A) \nsubseteq O(f(x_0)) \times O(g(y_0))$. Fix a point $(x', y') \in O(x_0) \times O(y_0) \cap A$ such that $(f(x'), g(y')) \notin O(f(x_0)) \times O(g(y_0))$. Obviously, $f(x') \in O(f(x_0))$. And since the map $g|_{\cup\{Y_\gamma : \gamma_0 \leq \gamma < sc(g)\}} : \cup\{Y_\gamma : \gamma_0 \leq \gamma < sc(g)\} \to T$ is continuous at every point of the set Y_{γ_0}, the condition $(f(x'), g(y')) \notin O(f(x_0)) \times O(g(y_0))$ holds only if the point $y' \in Y_\gamma$, where $\gamma < \gamma_0$. But this contradicts the choice of the set Y_{γ_0}, where $\gamma_0 = min\{\gamma(x) : x \in X_0 \cap A_1\}$. Thus, every point of the set $X_0 \times Y_{\gamma_0} \cap A$ is a continuity point of the restriction $f \times g|_A$.

The "only if" part. Scattered continuity of the maps $f : X \to S$ and $g : Y \to T$ follows from Proposition 3.2. Due to Proposition 2.1, there are indices $sc(f)$ and $sc(g)$ of scattered continuity of the maps f and g respectively and a pairwise disjoint families $\{X_\alpha\}_{\alpha < sc(f)}$ and $\{Y_\gamma\}_{\gamma < sc(g)}$ of non-empty subsets of X and Y respectively such that $X = \cup\{X_\alpha : \alpha < sc(f)\}$, $Y = \cup\{Y_\gamma : \gamma < sc(g)\}$; for each $\alpha < sc(f)$ the set X_α is dense in $\cup\{X_\beta : \alpha \leq \beta < sc(f)\}$ and $C(f|_{\cup\{X_\beta : \alpha \leq \beta < sc(f)\}}) = X_\alpha$ and for each $\gamma < sc(g)$ the set Y_γ is dense in $\cup\{Y_\lambda : \gamma \leq \lambda < sc(g)\}$ and $C(g|_{\cup\{Y_\lambda : \gamma \leq \lambda < sc(g)\}}) = Y_\gamma$. Suppose that both of these maps are not weakly discontinuous. Then there are some α_0, β_0 such that $X_{\alpha_0} \neq Int_{\cup\{X_\alpha : \alpha_0 \leq \alpha < sc(f)\}} X_{\alpha_0}$ and $Y_{\beta_0} \neq Int_{\cup\{Y_\beta : \beta_0 \leq \beta < sc(g)\}} Y_{\beta_0}$. Without loss of generality we can assume that $\alpha_0 = \beta_0 = 0$. Consider the subset $A = ((X_0 \backslash Int X_0) \times Y_1) \cup (X_1 \times (Y_0 \backslash Int Y_0))$ of $X \times Y$. Since the map $f \times g$ is scatteredly continuous, there is the point $(x_0, y_0) \in A$, the continuity point of the restriction $f \times g|_A$. Two cases are possible: $x_0 \in X_0 \backslash Int X_0, y_0 \in Y_1$ or $x_0 \in X_1, y_0 \in Y_0 \backslash Int Y_0$. Assume that $x_0 \in X_0 \backslash Int X_0, y_0 \in Y_1$. Since $y_0 \notin Y_0$, there is a neighborhood $O(g(y_0)) = O'$ of the point $g(y_0)$ such that for an arbitrary neighborhood $O(y_0)$ of the point y_0 we have that $g(O(y_0)) \nsubseteq O'$. Since (x_0, y_0) is a continuity point of the restriction $f \times g|_A$, for the neighborhood $O(f \times g(x_0, y_0)) = S \times O'$ there is a neighborhood $O(x_0, y_0) = O_1 \times O_2$ of the point (x_0, y_0) in $X \times Y$ such that $f \times g(O_1 \times O_2 \cap A) \subset S \times O'$. Since Y_0 is dense subset of Y, $O_2 \cap Y_0 \neq \varnothing$. We prove that there is the point $y' \in O_2 \cap Y_0$ such that $g(y') \notin O'$. Suppose this is not true. Suppose that $g(O_2 \cap Y_0) \subset O'$. Since y_0 is a continuity point of the restriction $g|_{Y \backslash Y_0}$, there is a neighborhood O_2' of the point y_0 such that $g(O_2' \cap (Y \backslash Y_0)) \subset O'$. Obviously, $g(O_2 \cap O_2') \subset O'$. And this contradicts the fact that $y_0 \notin Y_0$. Hence, there is a point $y' \in O_2 \cap Y_0$ such that $g(y') \notin O'$.

Since $x_0 \notin Int X_0$, for an arbitrary neighborhood $O(x_0)$ of the point x_0 we have that $O(x_0) \nsubseteq X_0$. Take some point $x' \in X_1 \cap (O_1 \backslash X_0)$. The point $(x', y') \in O_1 \times O_2 \cap A$, but $f \times g(x', y') \notin S \times O'$, which is a contradiction. □

Lemma 3.2. *Suppose we are given two finite families $\{X_i\}_{i \in \overline{1,k}}$ and $\{Y_i\}_{i \in \overline{1,k}}$ of topological spaces and a family of maps $\{f_i\}_{i \in \overline{1,k}}$, where $f_i : X_i \to Y_i$. The map $\prod\limits_{i=1}^{k} f_i : \prod\limits_{i=1}^{k} X_i \to \prod\limits_{i=1}^{k} Y_i$ is weakly discontinuous if and only if for each $i \in \overline{1,k}$ the map $f_i : X_i \to Y_i$ is weakly discontinuous.*

Proof. The "only if" part follows from Proposition 3.2.

Since the Cartesian product of topological spaces is an associative operation (see [4]) it is sufficient to prove the "if" part for two weakly discontinuous maps $f : X \to S$ and $g : Y \to T$. Since the maps f and g are both weakly discontinuous, by Proposition 2.2, there are ordinal numbers $wd(f)$ and $wd(g)$ respectively and a pairwise disjoint families $\{X_\alpha\}_{\alpha < wd(f)}$ and $\{Y_\gamma\}_{\gamma < wd(g)}$ of non-empty subsets of X and Y respectively, such that the spaces $X = \cup\{X_\alpha : \alpha < wd(f)\}$ and $Y = \cup\{Y_\gamma : \gamma < wd(g)\}$; for arbitrary $\alpha < wd(f)$ the set X_α is an open dense subset of $\cup\{X_\beta : \alpha \leq \beta < wd(f)\}$ and $Int_{\cup\{X_\beta : \alpha \leq \beta < wd(f)\}} C(f|_{\cup\{X_\beta : \alpha \leq \beta < wd(f)\}}) = X_\alpha$ and for each $\gamma < wd(g)$ the set Y_γ is an open dense subset of $\cup\{Y_\lambda : \gamma \leq \lambda < wd(g)\}$ and $Int_{\cup\{Y_\lambda : \gamma \leq \lambda < wd(g)\}} C(g|_{\cup\{Y_\lambda : \gamma \leq \lambda < wd(g)\}}) = Y_\gamma$. Analogously to Lemma 3.1 one can prove that the restriction $f \times g|_A$ is continuous at every point of the set $X_0 \times Y_{\gamma_0} \cap A$ (where γ_0 is determined as in Lemma 3.1).

We prove that $X_0 \times Y_{\gamma_0} \cap A$ is an open subset of A. We show that for any point $(x_0, y_0) \in X_0 \times Y_{\gamma_0} \cap A$ there is a neighborhood $O(x_0, y_0) = O_1 \times O_2$ of the point (x_0, y_0) such that $O(x_0, y_0) \cap A \subset X_0 \times Y_{\gamma_0} \cap A$. Since the set X_0 is open in X, there is the neighborhood O_1 of the point x in X such that $O_1 \subset X_0$. And since $\gamma_0 = min\{\gamma(x) : x \in X_0 \cap A_1\}$, for any neighborhood O_2 of the point y_0 in Y we have that $\pi_Y(O_1 \times O_2 \cap A) \cap Y_\gamma = \varnothing$ for each $\gamma < \gamma_0$. It remains to show that there is a neighborhood O_2 of the point y_0 in Y such that

$O_2 \cap Y_\gamma = \varnothing$ for all $\gamma > \gamma_0$. Suppose this is not true. Assume that for any neighborhood O_2 of the point y_0 in Y there is $\gamma > \gamma_0$ such that $O_2 \cap Y_\gamma \neq \varnothing$. Since the set Y_{γ_0} is open in $\cup\{Y_\gamma : \gamma_0 \leq \gamma < wd(g)\}$, each neighborhood $W \subset Y_{\gamma_0}$ of the point y_0 in Y_{γ_0} is an open subset of $\cup\{Y_\gamma : \gamma_0 \leq \gamma < wd(g)\}$. Then there is a neighborhood O_2 of the point y_0 in Y such that $W = O_2 \cap (\cup\{Y_\gamma : \gamma_0 \leq \gamma < wd(g)\}) \subset Y_{\gamma_0}$. Then $(O_2 \cap (\cup\{Y_\gamma : \gamma_0 \leq \gamma < wd(g)\})) \backslash Y_{\gamma_0} = \varnothing$, which is a contradiction. □

Theorem 2. *Let $\mathcal{F} = \{f_\alpha\}_{\alpha\in S}$ be a family of maps f_α of a topological space X_α into a topological space Y_α respectively. The Cartesian product $\prod\limits_{\alpha\in S} f_\alpha : \prod\limits_{\alpha\in S} X_\alpha \to \prod\limits_{\alpha\in S} Y_\alpha$ is a scatteredly continuous map if and only if the following conditions hold:*

(i) all the maps f_α are scatteredly continuous;

(ii) all the maps f_α, except maybe one, are weakly discontinuous;

(iii)all the maps f_α, except maybe finite number, are continuous.

Proof. The "only if" part. Statement (i) follows from Proposition 3.2 and statement (ii) follows from Lemma 3.1.

Suppose that the condition (iii) does not hold. We fix some countable infinite set $M \subset S$ of indices such that for each $\alpha \in M$ the map $f_\alpha : X_\alpha \to Y_\alpha$ is discontinuous. For every $\alpha \in M$ we fix a point $x^*_\alpha \in X_\alpha$ such that the map f_α is discontinuous at x^*_α. For each $\alpha \in M$ fix a neighborhood W^*_α of the point $f_\alpha(x^*_\alpha)$ such that for an arbitrary neighborhood $O(x^*_\alpha)$ of x^*_α we have that $f_\alpha(O(x^*_\alpha)) \backslash W^*_\alpha \neq \varnothing$.

Consider the set $B = \{\{x_\alpha\}_{\alpha\in S} \in \prod\limits_{\alpha\in S} X_\alpha$: there is $\alpha \in M$ such that $x_\alpha = x^*_\alpha\}$. Since the map $\prod\limits_{\alpha\in S} f_\alpha$ is scatteredly continuous, there is a point $x = \{x_\alpha\}_{\alpha\in S} \in B$ such that the restriction $\prod\limits_{\alpha\in S} f_\alpha|_B$ is continuous at x. Since $x = \{x_\alpha\}_{\alpha\in S} \in B$, there is $\alpha' \in M$ such that $x_{\alpha'} = x^*_{\alpha'}$.

We consider the neighborhood $V = \prod\limits_{\alpha\in S} V_\alpha$ of the point $\{f_\alpha(x)\}_{\alpha\in S}$ in $\prod\limits_{\alpha\in S} Y_\alpha$ such that $V_\alpha = Y_\alpha$ for all $\alpha \in S\backslash\{\alpha'\}$ and $V_{\alpha'} = W^*_{\alpha'}$. There is a neighborhood $U = \prod\limits_{\alpha\in S} U_\alpha$ of the point x in $\prod\limits_{\alpha\in S} X_\alpha$ such that $\prod\limits_{\alpha\in S} f_\alpha(U \cap B) \subset V$. Without loss of generality we can assume that $U = \prod\limits_{\alpha\in S} U_\alpha$ is an element of the base of the space $\prod\limits_{\alpha\in S} X_\alpha$, that is, there is a finite set $S' \subset S$ of indices such that $U_\alpha = X_\alpha$ for all $\alpha \in S\backslash S'$ and U_α is an open subset of X_α for all $\alpha \in S'$.

Since M is an infinite set, there is $\alpha'' \in M\backslash S'$ such that $\alpha'' \neq \alpha'$. And this means that $\pi_{X_{\alpha'}}(B \cap U) = \pi_{X_{\alpha'}}(\{\{x_\alpha\}_{\alpha\in S} : x_\alpha \in U_\alpha$ for all $\alpha \neq \alpha''$ and $x_{\alpha''} = x^*_{\alpha''}\}) = U_{\alpha'}$. Then $f_{\alpha'}(U_{\alpha'}) \subset \pi_{Y_{\alpha'}}(\prod\limits_{\alpha\in S} f_\alpha(B \cap U)) \subset \pi_{Y_{\alpha'}}(V) = W^*_{\alpha'}$.

The "only if" part. It follows from Lemmas 3.1 and 3.2, and from the fact that the Cartesian product of continuous maps is a continuous map. □

Corollary 3.1. *Let f be a map from a topological space X into a topological space Y. Then following conditions are equivalent:*

1. *The map $f : X \to Y$ is weakly discontinuous.*
2. *The Cartesian product $f \times f : X \times X \to Y \times Y$ is a scatteredly continuous map.*
3. *The Cartesian product $f \times f : X \times X \to Y \times Y$ is a weakly discontinuous map.*
4. *The Cartesian product $f^n : X^n \to Y^n$ is a weakly discontinuous map for each $n \in \omega$.*

Theorem 3. *Let $\mathcal{F} = \{f_\alpha\}_{\alpha\in S}$ be a family of maps f_α of a topological space X_α into a topological space Y_α respectively. The Cartesian product $\prod\limits_{\alpha\in S} f_\alpha : \prod\limits_{\alpha\in S} X_\alpha \to \prod\limits_{\alpha\in S} Y_\alpha$ is a weakly discontinuous map if and only if the following conditions hold:*
(i) all the maps f_α are weakly discontinuous;
(ii)all the maps f_α, except maybe finite number, are continuous.

Proof. The "only if" part follows from Proposition 3.2 and Theorem 2, and the "if" part follows from Lemma 3.2. □

Definition 3.1. *Let $\mathcal{F} = \{f_\alpha\}_{\alpha\in S}$ be a family of maps f_α of a topological space X into a topological space Y_α respectively. We say that the family $\mathcal{F}$ is scatteredly continuous (pointwise discontinuous), if for each non-empty (closed) subspace $A \subset X$ there is a point $x \in A$ such that for all $\alpha \in S$ the restriction $f_\alpha|_A$ is continuous at x.*

Definition 3.2. *Let $\mathcal{F} = \{f_\alpha\}_{\alpha\in S}$ be a family of maps f_α of a topological space X into a topological space Y_α respectively. We say that the family $\mathcal{F}$ is weakly discontinuous, if for each non-empty subspace $A \subset X$ there is a non-empty open subset B of A such that for all $\alpha \in S$ the restriction $f_\alpha|_A$ is continuous at every point of B.*

Proposition 3.3. *Let $\mathcal{F} = \{f_\alpha\}_{\alpha\in S}$ be a family of maps f_α of a topological space X into a topological space Y_α respectively. The diagonal product $\mathop{\triangle}\limits_{\alpha\in S} f_\alpha : X \to \prod\limits_{\alpha\in S} Y_\alpha$ is a pointwise discontinuous (scatteredly continuous, weakly discontinuous) map if and only if the family $\mathcal{F}$ is poinwise discontinuous (scatteredly continuous, weakly discontinuous).*

Proof. It is sufficient to show that $C(\mathop{\triangle}\limits_{\alpha\in S} f_\alpha|_A) = \bigcap\limits_{\alpha\in S} C(f_\alpha|_A)$ for each non-empty subspace A of X. An inclusion $C(\mathop{\triangle}\limits_{\alpha\in S} f_\alpha|_A) \subset \bigcap\limits_{\alpha\in S} C(f_\alpha|_A)$ follows from the equality $f_\alpha = \pi_{Y_\alpha} \circ (\mathop{\triangle}\limits_{\alpha\in S} f_\alpha)$ (which holds for all $\alpha \in S$).

It remains to show that $\bigcap\limits_{\alpha\in S} C(f_\alpha|_A) \subset C(\mathop{\triangle}\limits_{\alpha\in S} f_\alpha|_A)$. Take a point $x \in \bigcap\limits_{\alpha\in S} C(f_\alpha|_A)$ and show that the map $\mathop{\triangle}\limits_{\alpha\in S} f_\alpha|_A$ is continuous at x. Let $f_\alpha(x) = y_\alpha$ for all $\alpha \in S$. Assume that V is an arbitrary open subset of $\prod\limits_{\alpha\in S} Y_\alpha$ such that $\{y_\alpha\}_{\alpha\in S} \in V$. Then there is an element $\prod\limits_{\alpha\in S} V_\alpha$ of the base of space $\prod\limits_{\alpha\in S} Y_\alpha$ such that $\{y_\alpha\}_{\alpha\in S} \in \prod\limits_{\alpha\in S} V_\alpha \subset V$, $V_\alpha = Y_\alpha$ for all $\alpha \in S$, except finite number of indices $\alpha_1, ..., \alpha_k$, and V_{α_i} is open subset of Y_{α_i}, $i = \overline{1,k}$. Since for every $i \in \overline{1,k}$ the map $f_{\alpha_i}|_A$ is continuous at the point x, for the neighborhood V_{α_i} of the point y_{α_i} in the space

Y_{α_i} there is a neighborhood O_{α_i} of the point x such that $f_{\alpha_i}(O_{\alpha_i} \cap A) \subset V_{\alpha_i}$. Then $\bigcap_{i=1}^{k} O_{\alpha_i} \cap A$ is an open subset of A which contains x and $\underset{\alpha\in S}{\triangle} f_\alpha(\bigcap_{i=1}^{k} O_{\alpha_i} \cap A) \subset \bigcap_{i=1}^{k} \underset{\alpha\in S}{\triangle} f_\alpha(O_{\alpha_i} \cap A) \subset \bigcap_{i=1}^{k}(\pi^{-1}_{Y_{\alpha_i}}[\pi_{Y_{\alpha_i}} \circ \underset{\alpha\in S}{\triangle} f_\alpha(O_{\alpha_i} \cap A)]) = \bigcap_{i=1}^{k}(\pi^{-1}_{Y_{\alpha_i}}[f_{\alpha_i}(O_{\alpha_i} \cap A)]) \subset \bigcap_{i=1}^{k} \pi^{-1}_{Y_{\alpha_i}}(V_{\alpha_i}) = \prod_{\alpha\in S} V_\alpha \subset V$. Hence, the map $\underset{\alpha\in S}{\triangle} f_\alpha|_A$ is continuous at the point x. □

Since each finite family of weakly discontinuous maps is a weakly discontinuous family of maps, Proposition 3.3 yields the following.

Corollary 3.2. *A diagonal product of finite number of weakly discontinuous maps is a weakly discontinuous map.*

We show that the diagonal product of countably many of weakly discontinuous maps can be everywhere discontinuous.

Example 7. *Suppose X is the set $\mathbb{Q} \cap [0,1]$ equipped with the standard topology τ. For each $i \in \omega$ by X_i we denote the set $\mathbb{Q} \cap [0,1]$ endowed with the topology generated by the base $\tau \cup \{\frac{m-1}{i} : m \in \overline{1, i+1}\}$. Obviously, for each $i \in \omega$ an identity map $f_i : X \to X_i$ is weakly discontinuous. But the map $\underset{i\in\omega}{\triangle} f_i$ is everywhere discontinuous.*

Proposition 3.4. *Let $\mathcal{F} = \{f_i\}_{i\in\omega}$ be a countable family of weakly discontinuous maps f_i of a hereditary Baire space X into a topological space Y_i respectively. Then the diagonal product $\underset{i\in\omega}{\triangle} f_i : X \to \prod_{i\in\omega} Y_i$ is a pointwise discontinuous map.*

Proof. Suppose that for each $i \in \omega$ the map $f_i : X \to Y_i$ is weakly discontinuous. Let A be an arbitrary closed subspace of X. Since the map f_i is weakly discontinuous, the set U_i, of the points of continuity of the restriction $f_i|_A$, is open dense subset of A for all $i \in \omega$. Since the space X is hereditary Baire, the set $\bigcap_{i\in\omega} U_i$ is non-empty set of the points of continuity of the restriction $\underset{i\in\omega}{\triangle} f_i|_A : A \to \prod_{i\in\omega} Y_i$. □

The next example shows that the diagonal product of two scatteredly continuous maps from a compact space to Hausdorff space can be everywhere discontinuous.

Example 8. *Consider an identity maps $\varphi_1 : I \to I_{\mathbb{Q}}$ and $\varphi_2 : I \to I_{\mathbb{R}\setminus\mathbb{Q}}$ from the segment $I = [0,1]$ equipped with the standard topology τ to the segment $[0,1]$ endowed with the topology generated by the subbases $\tau \cup \{\mathbb{Q}\}$ and $\tau \cup \{\mathbb{R}\setminus\mathbb{Q}\}$ respectively. Obviously, both of these maps are scatteredly continuous, but the diagonal product $\varphi_1 \triangle \varphi_2$ is everywhere discontinuous.*

REFERENCES

1. Arkhangel'skii A.V., Bokalo B.M. *The tangency of topologies and tangential properties of topological spaces*, Trudy Moskov. Mat. Obshch., **54** (1992), 160–185, 278–279 (in Russian).

2. Baire R. *Sur les fonctions de variables reelles*, Annali di Mat., **3** (1899), 1–123.

3. Banakh T., Bokalo B. *On scatteredly continuous maps between topological spaces*, Topology and Appl., **157** (2010), 108–122.

4. Engelking R. General Topology, PWN, Warzawa, 1977.

5. Kannan V., Rajagopalan M. *Scattered spaces II*, Illinois J. Math., **21** (1977), 735–751.

6. Vinokurov V.A. *Strong regularizability of discontinuous functions*, Dokl. Akad. Nauk SSSR, **281**, 2 (1985), 265–269 (in Russian).

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Відображення $f : X \to Y$ між топологічними просторами називають розріджено неперервним (точково розривним), якщо для кожного непорожнього (замкненого) підпростору $A \subset X$ звуження $f|_A$ має точку неперервності. Відображення $f : X \to Y$ називають слабко розривним, якщо для кожного непорожнього підпростору $A \subset X$ множина $D(f|_A)$ точок розриву звуження $f|_A$ є ніде не щільною в A.

В роботі ми розглядаємо композицію, декартів і діагональний добуток слабко розривних, розріджено неперервних і точково розривних відображень.

Бокало Б.М., Колос Н.М. *Операции над некоторыми классами разрывных отображений* // Карпатские математические публикации. — 2011. — Т.3, №2. — С. 36–48.

Отображение $f : X \to Y$ между топологическими пространствами называют разрежено непрерывным (точечно разрывным), если для каждого непустого (замкнутого) подпространства $A \subset X$ сужение $f|_A$ имеет точку непрерывности. Отображение $f : X \to Y$ называют слабо разрывным, если для каждого непустого подпространства $A \subset X$ множество $D(f|_A)$ точек разрыва сужения $f|_A$ нигде не плотно в A.

В статье ми рассматриваем композицию, декартово и диагональное произведение слабо разрывных, разрежено непрерывных и точечно разрывних отображений.